De Numeris Geometricis.

OF THE NATVRE AND PROPRIETIES OF GEOMETRICALL NVMBERS.

Firſt written by *Lazarus Schonerus*, and now Engliſhed,
enlarged and illuſtrated with diuers and ſundry Ta-
bles and obſeruations concerning the mea-
ſuring of Plaines and Solids:

*All teaching the fabricke, demonſtration and vſe of a ſin-
gular Inſtrument, or Rular, long ſince in-
uented and perfitted by*
THOMAS BEDWELL Eſquire.

LONDON,
Printed by RICHARD FIELD.
1614.

TO THE HONORABLE KNIGHT,

Sir Robert Coke.

HONORABLE SIR,

HIſtorians report, that the Scythians were ſo far out of loue with learning, that it was a ſhame and diſgrace for any man amongſt them to be eſteemed a ſcholler. The name of a Scythian, all Chriſtians cannot chooſe but deteſt and abhorre. And yet is this their opinion, as we heare, maintained by the Nobilitie of ſome Nations. An argument, as I thinke, ſufficient alone, if hiſtorians were ſilent, to ſhew from what race ſuch men are deſcended. Barbarous, I am ſure it is, & no way conſonant to the nature of man. For as the Philoſopher ſaith, *Omnes homines naturâ ſcire deſiderant*. And, *Prudentia* (ſaith the Orator) *quæ in veri cognitione conſiſtit, maximè naturam attingit humanam*. The Nobilitie therefore of

England, are in this moſt highly to be commended, who do generally ſo much eſteeme of learning, that they thinke nothing more honorable then either to be accounted learned, or great patrons of learning: Knowing indeed that, as the Poet ſaith, *In prudentia Nobilitas ſita eſt.* This generallitie firſt; and then next, your honorable affection and fauour to me in particular, hath emboldened me to offer this ſmall Pamphlet vnto your patronage, *vt aliquod ſaltem extaret meæ in te obſeruantiæ & gratitudinis monumentum. Vale, V. O. ac nos tibi iampridem deuinctiſsimos, deuinctiores, ſi fieri poteſt, reddito. Londini Anglorum viij Kalend. Martias, an.* CIↃIↃCXIIII.

Your H. in all true affection,

William Bedwell.

To

TO THE COVRTEOVS READER.

IT is now, as farre as I remember, well neare fortie yeares ſince a Rular, for the meaſuring of Plaines and Solids, was firſt deuiſed by mine vnkle M. Thomas Bedwell. The vſe of this, himſelfe hath taught in a litle Treatiſe, which I haue ſeene long ago in his ſtudie amongſt other his writings of like nature and argumēt. The fabrick or maner of making it, he did not, to my knowledge, impart to any. That I ſuppoſe be reſerued for a larger volume of diuerſe other ſuch inuentions of his, which he had purpoſed to haue publiſhed, if God had ſpared him longer life. But theſe all being ſuppreſſed by ſome, who haue, as it ſeemeth, more reſpected their owne priuate gaine, then either his honour or the common good; I did, about a dozen yeares ago, as ſome learned louers of theſe ſtudies can teſtifie, labour to ſhadow out that worke of his. And at laſt hauing finiſhed a Treatiſe, both of the conſtruction and vſe of the ſame, I offered it vnto ſome to haue it imprinted: But they ſeeing in the copie *diuers figures and diagrammes, refuſed to meddle with it, except my ſelfe would beare a great part of the charge of that firſt impreſſion. That I being vnwilling to do, I was forced to deferre the publiſhing of it vntill I might meete with ſome one that would of himſelfe vndertake the buſineſſe. In this* interim, *I found yet another ſcruple, which I thought neceſſarie firſt to be remoued. For as the conſtruction of the firſt Author was meerely* Geometricall, *ſo that of ours was wholly Arithmeticall, and could not be done without the helpe of numbers: and therefore was ſuch as might not eaſily be taught or conceiued, except firſt the nature and proprieties of ſuch like numbers were well vnderſtood. Whereupon, before that Treatiſe of the Rular, this of* GEOMETRICALL NVMBERS *was firſt to be publiſhed. Geometricall numbers I call them; not* Figurate *or* Coſsicke *numbers, as commonly they are called: For of thoſe infinite ſorts of this kinde of numbers, we haue to do with none but ſuch as do repreſent and expreſſe ſome true* Geometricall *figure, ſuch as the* Quadrate, Parallelogram oblong, Priſma, *and the* Cubicke *are. Now whereas haply ſome man may obiect and ſay, that theſe things haue bene often handled and ſufficiently taught by others; let him know, that although in generall we do confeſſe it to be true, yet no man, to our knowledge,*

ledge, hath applied thoſe rules to that vſe that we haue done: And therefore thoſe Treatiſes of theirs, do in no waies fit our purpoſe. For firſt out of theſe rules, as afterward ſhall appeare, we haue framed certaine Tables for ſundrie vſes in meaſuring of Plaines and Solids. One of theſe, called TRIGONVM ARCHITECTONICVM, The Carpenters Squire, *we cauſed for the publicke good, more then a twelue-moneth ſince, to be imprinted at our owne coſt and charges. Moreouer, ſundrie practiſes and workes of theſe rules more plaine and eaſie then commonly are vſed, are here ſhewed and diſcouered. Laſtly, we haue here giuen thee a taſte or ſample of a forme and maner of reſoluing any intricate demonſtration mathematical, whereby it ſhall be made ſo plaine and eaſie, that it may be conceiued of the ſimpleſt. This we haue vſed theſe ſeuen or eight and twentie yeares; and becauſe we ſee it to be approued of the moſt learned Mathematicians of this kingdom, we ſhall ere long, God willing, more fully manifeſt it by many examples.*

OF A RATIONALL FIGVRATE NVMBER.

CHAP. I.

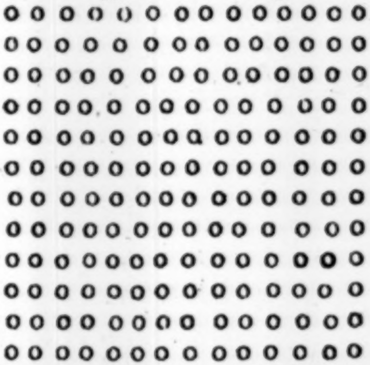
Of the Extraction of a Quadrate roote, or ſide.

1. ***A rationall figurate number is, a number that is made by the multiplication of numbers betweene them-ſelues.***

SVch a combination and affinitie there is between Arithmeticke and Geometry, that the whole nature & property of the one cannot well be taught and conceiued without the other. For ſo Ramus ſayth, *Geometria ſui & generis & iuris magnam quidem partem eſt, neque aliter quàm geometricè tractabilis. Attamen parte quadam numeris aſſociatur, & eis explicatur, numeriq́; geometricarũ affectionum interpretes, geometricis vocabulis appellantur, vt Planus, Quadratus, Solidus, & Cubus, à geometrico Plano, Quadrato, Solido, Cubo; quorum vmbræ quædam tales numeri ſunt. Itẽ, Ariſtoteles affirmat, generaliter arithmeticã demonſtrationem magnitudinum accidentibus conuenire, cũ magnitudines numeri fiunt: Et Proclus ait, Quicquid ip Geometria* ῥητὸν καὶ γνωστὸν *explicabile & cognobile ſit, numeris explicarì & cognoſci.* And indeed a great part of Euclides

Elements are wholly arithmeticall, that is, that booke of Euclide, whoſe purpoſe is to teach the generall grounds of Geometry, hath notwithſtanding many arithmeticall concluſions here and there intermedled among; and to ſpeake the truth, the vij, viij, ix, and x bookes of that his worke, are in a maner wholly arithmeticall. All this, I ſay, doth teath vs that Geometry connot be vnderſtood without the knowledge and vſe of Arithmeticke. Of this kinde moſt properly are thoſe figurate numbers, which do participate of both natures. And therfore one defineth a figurate number thus; *Figuratus numerus eſt numerus qui per figuras & appellationes geometricas exprimitur*, A figurate number is a number which is expreſſed by geometricall names and termes. Wherupon ſome haue called it a Geometricall number. Now a *Figure*, as Euclide ſayth, *is that which is contained on euery ſide with one or more bounds*, 14 d j. Such an one is a circle, contained with one line, which they call a Peripherie. Item, a Triangle, which is bounded with three lines: or a Quadrate, with foure lines. Such in bodies is a Cube, bounded with ſixe equall ſurfaces, & a Priſma, bounded with ſixe vnequall ſurfaces, &c. Now a *Rationall figure is a figure that is comprehended of the Baſe and Height rationall between themſelues*, 9 e iiij R: And the Baſe and Height are ſaid to be rational one to another, when as the rate or reaſon of both may be expreſſed by a number of the ſame meaſure giuen; by the 8 e j R. As for exãple, if the length of a quadrangle giuen be 14 inches, and the breadth 12, it is ſaid that that quadrangle is a rationall figure; becauſe the length, that is, the Baſe, and againe the breadth, or Height are both expreſſed by a number of the meaſure giuen, that is, by a certaine number of inches, to wit, that by 14 inches, this by 12. Now *to be comprehended of the*

Baſe and Height, is when the length is multiplied by the breadth. For this geometricall comprehenſiō, which here is vnderſtood, is as it were a multiplication by numbers. So in the former example the Quadrangle before named is comprehended of the Baſe 14, and of the Height 12. Therefore if thou ſhalt grant that the Baſe and Height are rationall betweene themſelues, or that their rate may be expreſſed by a number of the meaſure aſſigned: then I affirme, that the numbers of thoſe ſides being multiplied the one by the other, ſhall ſhew the content of that figure. Item that the product of them, that is, 168, is the figurate number expreſſing that content, as here thou ſeeſt.



2. *The numbers thus multiplied, or the numbers which make the figurate number, are called Sides or Rootes: and the arte whereby the Sides of a figurate number giuen are found, is called The extraction of a roote.*

That number which expreſſeth the *area*, or content of a rationall figure, is called, as we haue ſhewed before, a Figurate number: and the numbers repreſenting or expreſſing the Height and Baſe, that is, the numbers thus multiplied, or making this figurate, are by the geometricall terme called *Latera*, Sides: but vulgarly of the Arithmeticians they are called Rootes. For as plants and tres do ſpring from their rootes: ſo theſe figurate numbers, whether plaines or ſolides, do ariſe and haue their beginning from their rootes. If then the ſides be giuen, the figurate is eaſily found by multiplication. But to finde the ſides, rootes, or numbers wherof any figurate number was made, is a matter not ſo eaſie: That arte or rule that teacheth to performe this, is called, The extraction of a roote; of the Latines it is called, *Analyſis Lateris*: as the multiplication aforeſaid or manner of making a figurate number, is called of them *Geneſis figurati*.

3 *A figurate number is made by one multiplication, or by many: And either of them is equilater, or vnequilater.*

4 *An equilater figurate is made of equall numbers, or of one number multiplied by it ſelfe; which multiplier is alſo ſpecially called the Side or Roote: An vnequilater is that which is made of numbers which are vnequall betweene themſelues.*

As for example, the figurate 4, is made by one multiplication of one number by it ſelfe, to wit, of the ſide 2 by it ſelfe: Therefore 4 is a figurate equilater, and the ſide or roote of it is 2: So 9 is an equilater, whoſe ſide is 3. This ſide of the equilater by the Arabians is called *Radix*, that is, The Roote, as Schoner teſtifieth. Item 6 is a figurate of vnequall ſides, made, I ſay, of the multiplication of 2 by 3, and therefore 6 is an vnequilater figurate.

Here obſerue that an vnitie doth imitate euery kinde of equilater: For 1, by multiplication increaſeth not, neither yet doth it diminiſh any whit at all. It remaineth therefore therfore that an vnity multiplying an vnity, maketh but an vnity, that is, it taketh vpō it the nature of an equilater.

5 *Moreouer, an equilater figurate is twofold: either it is that whoſe true ſide is to be expreſſed by a number; or ſuch whoſe true ſide may not be expreſſed by any number.*

6 *The equilater whoſe ſide is to be expreſſed by a number, is that whoſe rate or reaſon vnto an* 1, *may be ſhewed.*

This diuiſion although it be not altogether proper to this place, yet becauſe it is commonly vſed by the vulgar ſort of Artiſts in this caſe, we would not omit it. The firſt ſort they call ῥητὸν, *effabile*, that is, ſuch as may truly be pronounced or ſpoken by ſome arithmeticall number: As for example, 16 is an equilater, or a figurate of equall ſides, and the true ſide is 4, that is, a number which may be ſpoken, vttered and ſet downe by arithmeticall figures; I meane whoſe quantity may be conceiued by the reaſon of it vnto 1. For in the quadrate or plaine equilater, as the reaſon of the number giuen is vnto the roote thereof: ſo is the ſame roote vnto an vnitie. For the roote or ſide is the meane proportionall between the figurate giuen, & 1. As in our example; as 16 is vnto 4, ſo is 4 vnto 1. And indeed al abſolute nūbers are conceiued & vnderſtood by the reaſon they beare vnto an vnity; and whatſoeuer they are, that they are ſayd to be in reſpect and compariſon of an vnity.

7 *The equilater whoſe true ſide cannot be expreſſed, is a figurate, the reaſon of whoſe ſide vnto* 1, *cannot be told or declared.*

As

As for example, 3 may be conceiued to be an Eequilater, that is, to be a product made of two equall numbers, or of one and the ſame number multiplied by it ſelfe, which number thus multiplied is, by the former, the ſide of the ſaid Figurate made. Now this ſide or number multiplied, is greater then 1. For 1, multiplied by it ſelf, by the 4 e, doth make the Equilater 1; which is leſſer then 3. Againe the ſame ſide is leſſer then 2. For 2 multiplied by 2, doth make 4, which is an Equilater greater then 3. Therefore the roote or ſide of 3, doth fall to be ſome meane quantity between 1, and 2. And yet what that number or difference is, or how ſpecially it is to be vnderſtood or conceiued, no man may poſſibly tell. Therfore ſuch a like ſide is called *arrheton, inexplicabile, irrationale*, Surde, irrational, or not to be vttered, as the Arithmeticians do call it.

8 *A figurate made by one multiplication, is called a Plaine: which is a figurate number made by the multiplication of two ſides equall one to the other.*

A figurate made by multiplication, as before is declared, is a number repreſenting a parallelogram, and yet not any indifferently, but that onely which is right-angled. Now *A Parallelogram is a Quadrangle whoſe oppoſite ſides are parallel*, *6 e x R*. And *A right-angled parallelogram is a parallelogram all whoſe angles are right-tangles*, 2 e x j R. And becauſe that all parallelograms are plaines; and for that plaines haue but two dimēſiōs, to wit Length & Breadth: therefore, by the 1 e, a plaine figurate is made by one multiplicatiō only; that is, by multiplying of the lēgth by the breadth. As for example, 9 is a figurate made of two ſides, to wit, of 3 multiplied by 3; and therefore 9 is a plaine figurate. Item 12 is a figurate made of two ſides; to wit, of 3 multiplyed by 4. Therefore 12 is plaine figurate, repreſenting a right-angled prarallelogramme ſurface.

Therefore

9 *If thou ſhalt deuide an inequilater parallelogramme by one ſide giuen, the quotient ſhall be the other ſide deſired.*

As for example, Suppoſe the figurate plaines were 12, and the one ſide thereof giuen were 3: here I ſay that the other ſide ſhalbe 4, to wit, the quotient of 12, diuided by the ſame 3. This is plaine by the 1 e 6 j lib. Arithmet. Salignaci. The argument is thus concluded:

If a number be made of two numbers giuen, the one of the numbers giuen ſhall deuide the product by the other: and contrariwiſe.

But a figurate plaine is made by the multiplication of two ſides one by the other: 8 e.

Ergo, The one ſide ſhall diuide the figurate plaine by the other.

This rule onely hath place in thoſe examples where the one ſide of the vnequilater plaine is giuē: that which followeth is more generall.

10 *All the rotes of the ſquares contained in a Figurate plaine giuen, which ſhall meaſure the ſaid figurate, with their quotients, ſhal be all the ſides of the ſaid plaine giuen.*

As for example; Let the figurate plaine giuē, al whoſe ſides are to be found, be 20: the ſquares contained in the ſame, whoſe ſides 1,2,4, do meaſure 20, let thē be 1,4,16: And the quotients by the ſame ſides, let thē be 20,10 &,5: I ſay that 1,2,4,5,10,20, are all the ſides meaſuring 20, the figurate plaine giuen. For although 9, be a quadrate alſo cōtained in the ſayd figurate; yet becauſe that 3, the ſide of the ſame, doth not meaſure the ſame figurate giuen, it is neglected as nothing pertaining to this our purpoſe. By this rule, as you ſee, may be performed that which the 2 verſe of the 13 Chapter of the firſt booke of Salignacus his Arithmeticke, doth teach; to wit, How to finde out all the meaſures of many, of any compound number giuen.

Let the compound number giuen, all whoſe meaſures of many are to be found, be 60: Here al the ſquares cōtained in 60, whoſe ſides may meaſure the ſame, are 1, 4, 9, 16, 25, 36: And the quotients of 60, by 1, 2, 3, 4, 5, 6, the ſides of the ſaid ſquares, are 60, 30, 20, 15, 12, 10: Therefore theſe twelue numbers 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60, are all the meaſures of 60, the compound number giuen.

11 *Like-plaines haue a doubled reaſon of their correſpondent ſides; and one meane proportionall comprehended of the extreme, or meane proportionall ſides.* 20 *p.* v j: *Item* 11 & 18 *p.* viij *E.*

Figurates haue their denomination of Figures, (as we haue ſhewed at the 1 e:) And therefore their nature cannot otherwiſe be conceiued & taught but from the ſame. Like figures therfore, as Ramus at the 14 e iiij, teacheth, are figures whoſe corners are equall and proportionall to the ſhankes of the equall angles. Now plaines haue but two dimenſions, and ſolides three. Wherefore they ſhall haue a doubled reaſon, theſe a trebled reaſon of the correſpondent ſides. Againe here, as Salignacus at the 2 e of the 14 Chap. of the ij booke of his Arithmeticke, teacheth, to double, treble, or quaduple a reaſon, is not to adde the ſame reaſon vnto it ſelfe twiſe, thriſe, or foure-times: But to multiply it by it ſelfe, twiſe, thriſe, or foure times. For example, let 8 and 18 be the two figurates giuen: And let their correſpondent ſides be 2 and 3: Item 4 and *6*: thoſe the *Baſes*, theſe the Heights. I ſay, that the reaſon of 8 vnto 18, is the reaſon of 2 vnto 3, or of 4 to *6*, doubled: The reaſon is thus doubled, $\frac{2}{3}\frac{2}{3}|\frac{4}{9}$: or thus, $\frac{4}{6}\frac{4}{6}|\frac{16}{36}$. The reaſon then of the firſt figurate vnto the ſecond, is the reaſon of 4 vnto *9*: or of 16 vnto 3*6*, that is, the reaſon of the firſt figurate vnto the ſecond, is *Subdupla-ſeſquiquarta.* For the conſequent being deuided by the antecedent, the quotient is $2\frac{1}{4}$. Therefore in Plaines, where the dimenſions are but two, to wit, Length and Breadth, the reaſon is onely doubled. This then is the reaſon of right-angled parallelogrammes, to which Plaine numbers do anſwer. And thus much of the firſt part of this propoſition: The ſecond part, of the meane proportionall, followeth. Like Plaines therefore, ſaith our Author, haue but one meane proportionall, comprehended of the proportionall ſides. The cauſe is vnderſtood out of the grounds of Arithmeticke: For if the reaſon of a number vnto a number be but doubled, then by the rule and nature of numbers continually proportionall, there can poſſibly be but one meane proportionall betweene them. The rule of this inuention, is thus laid downe by Salignacus, at the 2 e 12 c ij booke of his Arithmeticke: *If hauing two reaſons giuen, the ſecond bound of the firſt reaſon, do multiply both the bounds of the firſt reaſon: and the firſt bound of the ſecond, do multiply the firſt bound of the firſt reaſon, the products ſhall be continually proportionall to the foure numbers giuen.* In the two plaines before mentioned, 8 and 18, the proportionall ſides, or the two reaſons giuen, were 2, 3: 4, *6*: The meane continually proportionall, according to this rule, is thus:

$$\begin{array}{cccc} 2 & 3 & 4 & 6 \\ & 3 & 4 & \\ \hline 8 & 12 & & 18 \end{array}$$

The meane plaine then continually proportionall, betweene the plaines giuen, is 12: that is, as 2 is vnto 3, ſo is 8 vnto 12: and as 4 is vnto 6, ſo is the ſame 12 vnto 18. This meane proportionall therefore now found, is made or comprehended of 4, the length of the firſt figurate, and of 3, the height of the ſecond: or contrariwiſe of 2, the height of the firſt figurate, and of 6, the length of the ſecond: For the product, is ſtill the ſame: Therefore

12 *If the ſides of the two like Plaines be giuen, the meane proportionall of the ſame Plaines is alſo giuen.*

This is manifeſt by the former: For there the meane proportionall was made either of the middle proportionals, or of the two extreames of the foure numbers giuen. As for example, let the two Plaines giuen be 12 and 48: the length of the firſt, let it be 4; of the ſecond, let it be 8: The height or breadth of the firſt, 3; of the ſecond, 6: The product of 3 by 8; or of 4 by 6, that is, 24, is the meane proportionall deſired. The example is thus:

$$\begin{array}{ccc} 3 & \times & 6 \\ 4 & & 8 \\ \hline 12 & 24 & 48 \end{array}$$

I ſay, as 12 is vnto 24, ſo is the ſame 24 vnto 48. For the reaſon in both is *ſubdupla*. Thus much in generall of Figurate rationall numbers, or Figurate plaines: it followeth now in particular of their ſeuerall kindes.

CHAP. II.

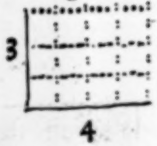
Of the Quadrate.

1 *An equilater or equal-ſided plaine, is called a Quadrate, 18 d vij, & 3 e xij R. The vnequal-ſided plaine is called an Oblong.*

OF the diuers ſorts of Plaines, handled by the Geometers, that onely is a Rationall plaine, all whoſe corners are equall, and oppoſite ſides parallell. Such an one is the Right-angled parallelogramme, which alone of all the Geometricall plaines is comprehended of the baſe and height, as before is ſhewed. Now the figurate of a rationall right-angled parallelogramme, is called a Plaine. Admit for examples ſake, that the Height of a right-angled parallelogramme were 3, and the Baſe were 4: Here if thou ſhalt multiply 4 by 3, the product 12, ſhall be the figurate, or content of the right-angled parallelogramme aſſigned. This product 12, is called a Plaine; and 3 and 4, the numbers whereof it was made, are called Sides, by a Geometricall name. And indeed this maner of multiplication, as we haue taught before, is meerly Geometricall. Place 12 vnities, (or, in ſtead of them, 12 ciphers) in equall diſtances one from another, as here thou ſeeſt; and it ſhall repreſent the figure, that is made of two right lines, whereof the one is 3 inches, the other 4. For if thou ſhalt firſt deuide the lines giuen into equall parts; the one into 3 parts, the other into 4: and againe ſhalt ſuppoſe, the Height to be erected perpendicularly vpō the end of the Length: And laſtly, conceiue firſt the Height to be caried or mooued all along from the one end of the Length vnto

o o o o
o o o o
o o o o

the other: And in like maner, the Length to be mooued vpward,for all that whole plumme-line : The traces that are to be ſuppoſed thoſe diuiſions will make vpon the plaine,ſhall make 12 ſquares,within the Oblong thus:



This kind of multiplication,I ſay,is geometricall: For here by this meanes, of lines are not made lines,(as there of vnities do ariſe vnities onely,) but a magnitude or bigneſſe,exceeding a line by one dimenſion,to wit,a ſurface. Before it was obſerued, that the figure thus made,is a Right-angled parallelogramme. Now a right-angled parallelogramme, is either an Oblong, or a Quadrate. An Oblong is a right-angled parallelogramme of vnequall ſides: A Quadrate or Square is a right-angled parallelogramme of equall ſides: 1 & 2 e xij: The figurate of a Quadrate is called alſo a Quadrate: which is a figurate made of two equall numbers multiplied betweene themſelues. This figurate aboue all other is to be accounted a Rationall:and yet euery Quadrate is not a Rationall number: For that onely is a Rationall, whoſe number is a true Quadrate;that is, ſuch a number is a rationall quadrate,whoſe ſide is to be expreſſed by a number. Of this kinde is that number onely that is made by the multiplication of one number by it ſelfe: c 3 e xij: Such are theſe nine following, made by the multiplication of the nine ſingle figures,or digittes,as they call them, betweene themſelues:

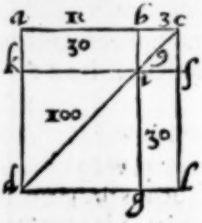
The Quadrates:	1	4	9	16	25	36	49	64	81
The Sides,or Rootes:	1	2	3	4	5	6	7	8	9

The Quadrate or Square number, is called of the Arabians,*Zenſus*: of the Greekes *Dynamis* a power or valour,as Euclide,Diophantus, and Barlaam do teſtifie. Wherefore whereſoeuer, in theſe artes, thou ſhalt meete with theſe phraſes, *Potentia recta eſt quadratum*, The power of a right line is a ſquare, or, *Potentia numeri eſt quadratus*, The power of a number is a Quadrate: there vnderſtand that any number giuē, multiplied in it ſelf, doth make a Quadrate. Item, *Si baſis trianguli ſubtendit rectum, æquè poteſt cruribus* : 5 e xij R. If the baſe of a triãgle be ouer againſt a right-corner, then the power of it is as much, as the power of both the other ſides: that is, if of the three ſides of a right-angled triangle thou ſhalt make three ſeuerall ſquares, the ſquare of that ſide ſhall be equall to the other two ſquares, made of thoſe ſides which do encloſe or containe the right-angle. Thus in Arithmeticke one number is ſaid to be able to do as much as two other numbers, whē as that number multiplied by it ſelf, ſhall make as much as thoſe two ſhal make, multiplied by themſelues : thus 10 multiplied by 10 maketh 100. Item *6* by *6*, giueth for the product *36* : And 8 by 8, yeeldeth *64*. Now becauſe that *36* × *64*, that is, *36* added to *64*, do make for the ſumme 100 : And that 100, are equall to 100; Therfore I ſay that the power of 10 is as much as the powers, or poſſibilities of *6*, and of 8.

This kinde of multiplication of a number by it ſelfe, is called *Quadratura numeri*, or *Geneſis quadrati*, the ſquaring of a number, or the making of a Quadrate : And contrariwiſe, the diuiſion of ſuch a like ſquare or quadrate, by the ſide whereof it was made, is called *Analyſis lateris quadrati*, The extraction or inuention of a ſquare roote. Now as euery Multiplication, and Diuiſion, ſo alſo this ſquaring of a number, and diuiſion or extraction of a ſquare roote, is done either iointly by the whole, or ſeuerally by parts, at many operations. The firſt is to be done and performed by Pythagoras table, which is ſet downe a little before: The ſecond is to be done by the preſcript of the Rule next following. But firſt that Table is to be learned by heart.

2 *If a number be diuided into two parts, the Square of the whole number ſhall be equall vnto the ſquares of the parts, with a double plaine made of them both.*

This in Geometrie, ſpoken of a right-line, is the 8 e xij R: and is thus laid downe by Ramus: *Si recta eſt ſecta in duo ſegmenta, quadratum totius æquatur quadratis ſegmentorum, cum duplici rectangulo vtriuſque;* If a right line be cut into two portions, the ſquare of the whole line, is equall to the ſquares of the portiõs, with two right-angled parallelogrammes made of both the ſaid portions. It is a conſectarie drawne out of the 3 c 11 e x booke of Ramus his Geometrie. Suppoſe the right-line giuen were a c: let it be in b, deuided into two parts, a b, and b c. Now let the quadrate made of the whole line a c, be a c l d: and from the diuiſion b, drawne a line parallell to the ſide c l: Item, at the ſame diſtance, draw another parallell to a c; and thou ſhalt deuide the quadrate a c l d, into two ſorts of right-angled parallelogrammes, to wit, into c i, and d i, two diagonals: Item, into a i, and l i, two complements, as the Geometers call them. Now that theſe two diagonals, with the two complements, are equall vnto a c l d, the quadrate made of the whole line a c, it is moſt manifeſt. For the whole is equall to all his parts ioyntly taken. But the two Diagonals, and two Complements aforeſaid, are all the parts of the Quadrate a c l d, as here thou ſeeſt. Therefore if a right-line be deuided &c. This rule the Arithmeticians, ſpeaking of numbers, doe ſet downe in theſe tearmes: If a number, &c. Which Barlaam hath



hath arithmetically demonſtrated;to whom I referre thee. Out of this rule is framed the Quadrature or Fabricke of the whole ſquare,by the two ſegments or portions of the ſide;and that in this maner: As for example, let the number giuen be 7, and the quadrate of the whole number let it be 49. Then deuide the ſame into two portions, to wit, into 4 and 3. And let theſe two numbers multiply one another,the foure products *9*,12,12, and 16 added, ſhall make 49, equall vnto the former product,made of *7*,the whole number. Againe, let the number giuen be 13; and the ſquare of this whole number let it be *169*. Now let 13 the whole line, be deuided into two parts, to wit,into 10 and 3:let theſe multiply one another, and the ſeuerall products ſhall be foure, the ſumme of which added together,ſhall be equall vnto the quadrate or product made of the whole line 13:the example is thus:

13 The whole number giuen.
13
169 The Quadrate of the whole.

10 3 The portions of the whole number deuided.
10 3

9 The Quadrate of the leſſer portion.
30 The two Plaines made of the two portions multiplied betweene themſelues.
30
100 The Quadrate of the greater portion.
169 The ſumme of the Quadrates and two Plaines, equall to the Quadrate of the whole.

Therefore

3 *The ſide of the greater Quadrate, is the ſide of one of the Plaines; and being doubled,it is the*

ſide of them both together: The other ſide of both the Plaines ioyntly taken, is the ſide of the leſſer Quadrate.

It followeth neceſſarily of the former. This rule teacheth how to reſolue a Quadrate giuen, or the maner of Extraction of a ſquare or quadrate roote. The former rule taught that the true ſquaring of a number, diuided into parts, conſiſted of two ſquares and two plaines. So that the true reſolution of a Quadrate ſo made, muſt be the inuention of the ſeuerall ſides of the ſaid particular ſquares and plaines, whereof the whole Quadrate giuen conſiſteth. For it is the ſame way from Cambridge to London, that it is from London to Cambridge. By the former, the ſide of the ſquare *169*, was 13: which ſide or number conſiſteth of two ſeueral numbers, to wit, of the article 10, and the digit 3. Here, by the former rule, the greater ſquare is 100, and the ſide thereof is 10: The one Complement or Plaine is 30, made of two ſides, whereof the one is 10, the ſide of the former ſquare: Therefore 30, is the ſide of *6*0, the ſumme of both the plaines. The other ſide of the Plaine (ſingle or double) is 3, which is alſo the ſide of *9*, the leſſer ſquare. Theſe grounds thus laid, the practiſe is to be performed according to the direction of this rule next following.

4 *If the ſide of ſome greater ſquare be ſought, firſt beginning at the right-hand, diſtinguiſh the number giuen continually by paires, for ſo many particular ſquares: Then ſetting downe the ſide of the firſt particular ſquare found within the Quotient, ſhalt double the ſide found,*

found, for the baſe of the firſt Complement. Laſtly, the Complement deuided by the Baſe, the Quotient ſhall be the ſide of the next ſucceeding Square.

As in 144, the example ſo oft mentioned, firſt beginning at the right-hand, or firſt figures, I diſtinguiſh it thus, 1,44; wherby I vnderſtand that the particular ſquares, whereof the whole ſide is comprehended, are two. This done, I ſeeke amongſt the ſquares of the ſingle figures, at the 1 e, for the ſide of 1, the firſt ſquare, & I finde the ſide to be alſo 1: This ſide I place in the quotient. This ſquare I ſubtract from 1, the laſt period, and there remaineth nothing. Secondarily, I double the ſame quotient, or ſide found, and I make 20 (for indeed the firſt diagonall's ſide is 10; and the ſide of both together iointly taken is 20; as we ſaw at the 3c.) By this I deuide 44, the whole period, or number remaining, and I finde the quotient to be 2, for the other ſide of the Complements, and the roote of the leſſer Diagonall: Therefore I multiplie firſt this quotient laſt found by it ſelf, and I make 4, for the leſſer Diagonall or ſquare. Againe I multiplie the ſame quotient by 20 the Deuiſor, and I make 40, for the doubled Complement. Laſtly, I ſubtract 44, the ſumme of the ſaid doubled complement, and leſſer diagonall, from 44, the number remaining, and there remaineth nothing: Therefore I ſay that, 144 the figurate giuen, is a true Quadrate; And the ſide or Roote thereof is 12, which was deſired.

Item, Let the ſide of the Quadrate 9604 be ſought. Firſt, beginning at the right hand, I diſtinguiſh the quadrate giuen continually into payres of degrees, thus 96,04; and I finde the number to containe two ſuch paires: And therfore I conclude that the whole ſide of the quadrate giuen doth conſiſt of two ſingle figures. This done, I firſt ſeeke amongſt the ſquares of the ſingle figures, at the 1 c, for the ſquare *96*; which number, becauſe I finde it not amongſt them, I ſay is no true ſquare. Now the greateſt ſquare conteined in *96* I finde to be 81, and the ſide or Roote of it to be *9*. This ſide I place within the quotient. Then multiplying *9* by it ſelfe, I make 81, for the greateſt Diagonall, which placed vnder *96*, and ſubtracted from the ſame, there do remaine ouer the head thereof 15: Now cancelling 04, I place them alſo ouer the head, as high as the ſaid 15. Secondarily, doubling *9* my quotient nowe found, I make 18 (or, for the reaſō before recited, 180) for the Baſe of the Complement: Then by this baſe I deuide 1504, the nūber remaining, and I find the quotient 8. This therfore I likewiſe ſet downe by *9*, for the other ſide of the Plaines, as alſo for the ſide of the leſſer Quadrate. *For the proofe of this later worke, I multiplie firſt the ſaid quotient by it ſelfe: Then I multiply the ſame quotient by the doubled Complement, that is, by the Deuiſor: Laſtly I ſubtract the ſumme of theſe two products from the number remaining.* As in this our example, I multiply 8, the quotient, by it ſelf, and I make *64*, for the ſecond or leſſer quadrate: Then I multiply 180, my deuiſour, or baſe of the complements, by the ſame quotient 8, and I make 1440, for the ſaid double Complement: Now 1440 × *6*4, or the ſumme of 1440 and *6*4, is 1504. This, I ſubduct from 1504, the number remaining, and there is left nothing: Therefore I ſay that *96*04, the number giuen, is a true Quadrate, and the ſide or Roote thereof is *9*8, which was deſired. The example is thus:

```
     1504
180)9604(98
     81
     1504
```

I. The baſe of the Complements:	II. The doubled Complement:	III. The leſſer Quadrate:	IV. The Summe:
90	180	8	1440
2	8	8	64
180	1440	64	1504

Therefore if thou ſhalt multiply 98 in it ſelfe, thou ſhalt make 9604, the number giuen. For the Quadrates of the Segments 90, and 8, with their Plaines or Complements, are the parts of the Quadrate of the whole number 98. The geneſis or making of this Quadrate, after our preſcript, is thus:

90 8
90 8
64
720
720
8100
9604

5 *Hauing found the Quotient of two, or more figures, if yet the whole ſide of the Quadrate giuẽ be not found, thou ſhalt double the whole quotient already found: And then ſhalt in all things elſe whatſoeuer, obſerue the ſame order as was preſcribed in the former.*

And in this example: The ſide of 15129 is deſired: Firſt, hauing deuided it, as in the former we haue taught, continually into paires, thus, 1,51,29, I finde 1 to be a Quadrate; and 1 alſo to be the ſide thereof. This ſide I write in the Quotient. The quadrate of this quotient I ſubtract from 1, and there remaineth nothing. Then I double the ſaid quotient 1, and I make 2, (which in reſpect of the next period I cal 20) for the baſe of the doubled Complement:

By this I deuide 51, the next period, and I find the quotient 2, for the height of the ſaid Complement, and ſide of the leſſer quadrate. This latter quotient therfore I multiply firſt in it ſelfe, and I make the quadrate 4: Then again by the ſame quotient I multiply my deuiſor 20, and I make 40, for the doubled complement: Now 40 ⨯ 4, are 44, which I ſubtract from 51, and there remaine 7: Therefore the doubled Complement, with the ſecond Quadrate, is 729. Laſtly by this rule, I double the whole quotient 12, and I make 24, or in reſpect of the period following 240, for the baſe of the doubled Cōplement: now the quotient of 729, by 240, is 3, for the ſide of the leſſer quadrate, and height of the ſaid Complements. This done I multiply 3, the laſt quotient found by it ſelf, and I make 9, for the leſſer quadrate. Item I multiply by the ſame quotient 3, the deuiſour 240, and I make 720, for the doubled Complement. Now 720 ⨯ 9, that is, 729, I ſubtract from 729, and there remaineth nothing. Therefore I conclude that 15129 the number giuen, is a quadrate: And the ſide thereof is 123, conſiſting of three figures. The whole worke is thus:

~~7~~ ~~2~~ 9
~~5~~ ~~1~~
20) ~~1~~, ~~5~~ ~~1~~, ~~2~~ 9 (123
~~1~~
~~4~~ ~~4~~
~~7~~ ~~2~~ 9

	I. The baſe of the Complement.	II. The doubled Complement.	III. The leſſer Quadrate	IV. The ſumme of the Complem. and Quadr.
I	1 2 20	20 2 40	2 2 4	40 4 44
II	12 2 240	240 3 270	3 3 9	720 9 729

In greater examples there is yet greater variety to be obſerued. Let therefore the practioner reſolue this one great Quadrate, *6192967290651525224*, ſuch as are often to be reſolued in the making of thoſe tables of *Sines, Secants*, and *Tangents*, of ſo great and wonderful vſe in many buſineſſes, where the vſe of theſe Artes are required.

From hence do follow many particular Conſectaries, Firſt the *Single figures of any Quadrate giuen cannot exceed the double of the number of the ſingle figures of the ſide thereof.* The reaſon is from the 1 e ij: to wit, becauſe the product of the greateſt ſingle figure by it ſelf, cōſiſteth not of more then two figures. And

2 *If the number giuen beginning at an vnity, and ſo increaſing according to the natural order of nūbers vnto the middlemoſt, ſhall from the ſame, in the ſame maner decreaſe vnto an vnity, the ſide of the ſayd quadrate ſhall conſiſt of vnities: And the ſame middle number ſhall ſhew the number of them.* As for example: Suppoſe the number giuen were 1234321, here becauſe it beginneth at an vnitie, and ſo increaſeth vnto the middlemoſt, and from the ſame middlemoſt decreaſeth in like maner continually vnto an vnity: And for that the ſame middlemoſt number is 4: I ſay that the ſide or roote of the quadrate giuen conſiſteth altogether of vnities: And that the number of them is but 4, thus 1111. Item, the ſide of this quadrate following, 12345678987654321, is 111111111. Therefore on the contrary, *If a number to be multiplied by it ſelfe, do conſiſt altogether of vnities, vnder the number of tenne, the product ſhall from an vnity increaſe and decreaſe, as aforeſayd.*

6 *The difference of two vnequal quadrates giuen, is the quadrate of the difference of their ſides, with a double plaine made of the ſame differēce,*

and the leſſer ſide. This is called the Gnomon, *or ſquire.* 2 *d* xij; *or* 12 *e* x R.

As for example, Let the two Quadrates giuen be 144, and 100; And their Difference let it be 44. Againe, Let their Sides be 12 and 10; And their difference let it be 2. Nowe let the Quadrate of 2, the Difference of the ſides be 4. Againe, Let the Plaine made of 10, the leſſer ſide, & of 2, the Difference of the ſides, be 20. Now 20 ⋈ 20, are 40. And 40 ⋈ 4, are 44, the Gnomon or difference, whereby the two Quadrates 144, and 100, do differ one from another. Therefore

7 *If the number giuen be not a true Quadrate, the remaine (to be added vnto the ſide of the greateſt Quadrate conteined in the ſayd number giuen) ſhalbe denominated of the Gnomon, or difference of that ſaid greateſt Quadrate, and that which is next aboue it.*

Let the number giuen be 148, which by the 5 *e*, is no true Quadrate: and therefore the true ſide thereof can neuer be found. But the ſide, ſomewhat neare vnto the true ſide, is thus found. The greateſt Quadrate contained in 148, the number giuen, 144, whoſe ſide by the ſaid 5 *e*, is 12. Item, the Remaine or difference of 144, & 148, is 4, for the Numerator of the parts ſought. Againe, the Quadrate next greater then 144, is *169*, and the ſide thereof is 13. Laſtly, the Gnomon or Difference betweene *169*, and 144, is 25, the denominator ſought. Therefore $\frac{4}{25}$, are the parts deſired. The ſide then of 148, neare vnto the true ſide, is 12$\frac{4}{25}$. Briefly,[1] *The Denominator of the parts ſought, is the difference of the greateſt ſquare contained in the num-*

ber giuen, and the next greater aboue it. Let the number giuen be 11: the greater Quadrate contained in it is 9, whoſe ſide is 3. Now 11 —9, is 2, for the Nummerator deſired. Item, the next Square greater then 9, or 11, is 16, and the ſide thereof is 4. Now 16—9, is 7, for the Denominator of the ſaid parts. The ſide of 11, neare vnto the true ſide, is, $3\frac{2}{7}$. Itẽ,[2] *The ſaid Denominator is the double of the ſide of the greateſt Quadrate encreaſed by an vnitie.* In the laſt example, the ſide of the greateſt quadrate contained in 11, is 3; whoſe double is 6. Therefore 6+1, that is 7, is the Denominator ſought, and the parts are $\frac{2}{7}$. Againe, [3] *The denominator of ſuch like parts ſhall be the ſumme of the ſide found, and of the ſide of the next greater quadrate aboue it.* In the former example, the ſide of 9, the greateſt Quadrate contained in 11, is 3: and the ſide of 16, the quadrate next aboue 9, is 4. Now 4 × 3, that is 7, is the Denominator deſired: and the ſide ſought is $3\frac{2}{7}$.

8 *The Product made of two like-plaines, is a Quadrate: and the ſide thereof is the product of the baſe of the one, by the height of the other.*

Like-plaines, are plaines whoſe ſides are proportionall, 20 d vij. Let the Like-plaines giuen be 6, and 24: whoſe ſides, 3, 2, 6, 4, are proportionall: that is, as 3, the baſe of 6, is vnto 2, the height of the ſame: ſo let 6, the baſe of 24, be vnto 4, the height thereof. Here firſt I ſay, that 144, the product of 24, by 6, is a quadrate. Againe I ſay, that 12, the product of 3, the baſe of 6, the firſt plaine, by 4, the height of 24, the ſecond plaine: or contrariwiſe, the product of 2, the height of 6, the firſt plaine; by 6, the baſe of 24, the ſecond plaine, ſhall be the roote or ſide of the ſaid Quadrate. The example is thus:

$$\begin{array}{r} 24 \\ 6 \\ \hline 144 \end{array} \qquad\qquad \begin{array}{ccc} 3 & & 6 \\ & \times & \\ 2 & & 4 \\ \hline & 12 & \end{array}$$

The Quadrate. The ſide or roote thereof.

9 *If three numbers be continually proportionall, the product of the firſt and the laſt ſhalbe equal to the quadrate of the middle numbers; And contrariwiſe, If the quadrate of the middle number be equall to the product of the firſt and the laſt, the three numbers giuen ſhalbe continually proportionall.*

Let the three numbers giuen continually proportionall, be 4,*6*,*9*, the product of 4,by *9*,the firſt and the laſt, ſhall be *36*: Item the ſquare or the product of *6* by it ſelfe ſhall alſo be *36*: On the contrary, if the quadrate, ſquare, or product of the middlemoſt be equall vnto the product of the firſt and the laſt, the three numbers giuen are continually proportionall: As Euclid doth demonſtrate and teach at the *16* p vj,and 20 p vij,of his Elements. Item, Ramus handleth this propoſition in three ſeuerall places; to wit at the 15 e ij of his Arithmeticke: At the ij conſect. of the 14 e x, and againe more ſpecially at the 4 e x ij, of his Geom. Yet in deed,ſaith Schoner, there is ſomething in this more then may be performed by that 15 chap. of the ſecond booke of his Arithmeticke. For diuiſion cannot alwayes finde out the meane proportionall: But in diuerſe caſes there is required,to the performance of this,the extraction of a ſquare roote (of which we haue ſpoken before at the 4 and 5 e:) by the which alſo the Deuiſour is ſought. As for example, ſuppoſe the meane proportionall betweene 16, and 64, the extreames giuen, were to be ſought. Here the product of 64 by 16, is 1024; and the ſide of 1024, by the 4 e, is 32, the meane proportionall deſired.

A proportion requireth foure bounds or numbers. If the ſecond and third bound be the ſame, as in this our example, it is called a Continuall proportion: If they be diuerſe, it is named a Diſioyned proportion. Now generally, whether the proportion be diſioyned or continuall, this rule is true: *If foure numbers be proportionall, the product of two middle numbers ſhalbe equal to the product of the firſt and the laſt.* As for example, As 12 is to 4, ſo let 6 be to 2. I ſay the product of 6 by 4, ſhalbe equall to the product of 12 by 2. For let the product of 12 by 6, be 72. Here therefore the product of 6 by 12, ſhalbe alſo 72, by the 3 e iiij c j Saligna.

- *If two numbers be proportional to one and the ſame number, they are equall betweene themſelues: 16 e v c j Salig.*
- *But 72 is proportionall vnto the product of 12 by 2: and againe, to the product of 6 by 4*
  - 1°
    - *If one number do multiply many numbers, the products ſhalbe proportionall vnto the numbers multiplied: 5 e vj.*
    - *But 12 doth multiply 6 (and maketh 72 by the conſtruction:) Item it multiplieth 2.*
    - *Therefore as 72 is vnto the product of 12 by 2: ſo is 6 vnto 2: and ſo, by the grant, is 12 vnto 4.*
  - 2°
    - *If one number do multiply many, &c. 5 e vj c j Salign.*
    - *But 6 doth multiply 12, and 4.*
    - *Therefore as 72 (which is the product of 6 by 12, as before was manifeſt) is vnto the product of 6 by 4, ſo is 12 vnto 4.*
- *Therefore the products of 12 by 2, and of 6 by 4, are equall. q.e.d.*

This propoſition is commonly called the *Golden rule*: and indeed truth it is that the Mathematical arts do daily reape from hence fruites in value more worth then gold.

10 *If the three numbers continually proportionall giuen, be ſimple between themſelues, the firſt & the laſt ſhalbe quadrate or ſquare numbers: Item, If of three numbers continuallie proportionall giuen, the firſt be a quadrate, the third alſo ſhalbe a quadrate.*

As for example, Let the three numbers continually proportionall, and ſimple betweene themſelues giuen, be 4, *6*, *9*; the firſt and the laſt, to wit 4 and *9*, are quadrate numbers.

Again, if 1*6*, 32, *6*4, be continually proportionall, & 1*6*, the firſt bound be a quadrate; *6*4 alſo, the third bound ſhalbe a quadrate. Theſe both are demõſtrated by Euclid, and his Expoſitours at the 2, and againe at the 22 prop. of his viij booke.

11 *If a quadrate do multiply a quadrate (or one many continually) the product ſhalbe a quadrate; and the ſide of the quadrate ſo made, ſhalbe the product made by a continuall multiplication of the ſeuerall ſides of the particular quadrates giuen.*

As for example; Let the quadrate 4, multiply the quadrate *9*, and let the product thus made be 3*6*: Here firſt I ſay,the product of 36 is alſo a quadrate.Item I ſay, that 6, the product of 2 by 3, the ſides of 4 & 9, the particular quadrates giuen,ſhalbe the ſide of 36,the compound quadrate.

Itē,25401600,the product continually made of 4,9,16, 25,36,49, is a compound quadrate: And the ſide thereof is 5040,which is the product made of 2, 3,4,5,6, 7, continually multiplied between them ſelues.

12 *If a Quadrate doe deuide a number aſsigned by a Quadrate, the aſsigned ſhall be a Quadrate.*

This alſo ariſeth from the ſame fountaine. As for example, Let 4, a quadrate deuide 64, the number aſſigned by 16, a quadrate. I ſay that 64, the number aſſigned is likewiſe a quadrate.

13 *If the product of two numbers aſsigned be a quadrate, the ſide of that quadrate ſhal be the meane proportionall betweene the aſsigned.*

As for example, Let the nuumber aſſigned, between which we deſire the meane proportionall, be the quadrates 144,and 64;and let the product of them be 9216: I ſay that 96,the ſide of the ſaid compound quadrate made, ſhalbe the meane proportionall deſired, that is, as 144 is vnto 96: So 96 is vnto 64. The demonſtration is built vpon the 9 e vj c j Salign.which is a ſpecial conſectary drawn from the 2 c 14 e,of the x booke of Ramus his Geometry: The argument is thus framed

If the product of the two middlemoſt numbers be equal to the product of the firſt and the laſt, the foure numbers giuen are proportionall betweene themſelues: 9 e vj c j Sal.

But the product of 144 by 64, is 9216 by the grant: And the product of 96 by 96, the 2 middle nũbers, is alſo 9216.

The ſide is a number, which multiplied in it ſelfe, doth make a quadrate.
But 96 is the ſide of 9216, by the conſtruction and grant.
Therefore 9216 is made of 96, multiplied by it ſelfe.

Therefore 144, 96, 96, 64, the foure numbers giuen, are proportionall betweene themſelues: and ſo the meane proportionall betweene 144, and 64, the two numbers giuen, is found, q.e.f.

Therefore

14 *If a number ſhall by it ſelfe deuide a nũber made of any two numbers giuen, the quotient ſhalbe the meane proportionall between the two aſſigned: 10 e vj c i Salig.*

The rule is generall thus: *If one number ſhall deuide the product of two aſſigned, the deuiſour and the quotient ſhallbe the meane continnally proportionall, betweene the aſſigned.* As for example, let *6*, multiplying 4, make 24: And let 8 deuide 24 by 3: I ſay, as *6* is to 8, ſo is 3 to 4. But the ſpeciall conſectarie is more fitter for our purpoſe. Therefore for example, Let the product 144 by *64*, be *9216*, and let *96* diuide *9216*, the ſaid product, by *96*: I ſay, that *96* is the meane proportionall betweene 144, and *64*, the two numbers giuen: that is, as 144 is vnto *96*, ſo is *96* vnto *64*.

CHAP.

CHAP. III.

Of the making and vſe of certaine Tables and Inſtruments, deuiſed for the more exact and ſpeedie meaſuring of all ſorts of Plaines.

1 PLaines, according to their diuers nature and qualitie, are meaſured by diuers and ſundry kindes of meaſures: For ſome are meaſured by the Foote, others by the Yard or Elne, otherſome by the Rodde or Perch, and ſuch like. Now theſe meaſures being defined by Act of Parliament, it ſhall not be amiſſe to ſet downe the words of the Statute, ſo farre foorth as it ſhall concerne this argument. It is ordained, ſaith the Statute, that three graines of barley drie and round, doe make an Inch; twelues inches do make a Foote; three foote do make a Yard; fiue yards and an halfe, doe make a Perch; and fortie Perches in length, and foure in breadth, doe make an Akre. 33. Edw. I, *De terris menſurandis*. Item, *De compoſitione Vlnarum & Perticarum*. See alſo more of this hereafter.

By the Foote we meaſure Boord and Glaſſe. A Foote therefore of flat meaſure, is a right-angled ſquare, 12 inches long, and 12 inches broade, that is, a foote of boord is a plaine, containing 144 ſquare inches: For ſuch is the product of 12 by 12. Here, as alſo in the other which follow, obſerue, That the Breadth is alwaies giuẽ; the Length is deſired. If therefore the breadth giuen be 12 inches, it is plaine by the former definition, that 12 inches of length doth make a foot of flat meaſure. But if the breadth giuen, be either greater or leſſer then 12, the length deſired is not ſo eaſily found. For here ſome art is oft times required. This then is to be conceiued and done by the

14 e of the former Chapter: For there we learned how to equall plaines of diuers breadths. Admit now a boord to be meaſured were but 9 inches broade; here, as thoſe rules did teach vs, I diuide 144, by 9 the breadth giuen, and I finde in the quotient *16*, for the length deſired. For as 12 is to *9*, ſo is *16* to 12; that is, *16* inches in length, of the breadth of *9*, are equall to 12 inches of length, of the breadth of 12. Thus you may make a Table or Inſtrument to ſerue readily at all times, for the more ſpeedie and exact meaſuring of all ſorts of plaines by the foote ſquare. For, *If you ſhall diuide* 144, *by the ſeuerall breadths giuen, the quotients ſhall ſhew the length deſired, anſwering to their ſeuerall diuiſors or breadths giuen.* Now where you do beginne (I meane whether at the greater breadth, and ſo deſcending ſhall end at the leſſer, or contrariwiſe) the matter is not great. Againe, the Carpenters or workemens inſtrument which they vſe in this caſe, being but two foote in length, it ſhall not be neceſſarie to beginne at any breadth greater then 24, as ſhall be manifeſt hereafter, when we come to ſhew the vſe of this Inſtrument or Rular. Beginning then at 24, and ſo deſcending to the leſſer, the Table is thus:

The

Br.	Le.	B.	L.	B.	L.
24	6	12	12	6	24
23	$6\frac{6}{23}$				
22	$6\frac{6}{11}$	11	$13\frac{1}{11}$		
21	$6\frac{6}{7}$				
20	$7\frac{1}{5}$	10	$14\frac{2}{5}$	5	$28\frac{4}{5}$
19	$7\frac{15}{19}$				
18	8	9	16		
17	$8\frac{8}{17}$				
16	9	8	18	4	36
15	$9\frac{3}{5}$				
14	$10\frac{2}{7}$	7	$20\frac{4}{7}$		
13	$11\frac{1}{13}$				
12	12	6	24	3	48
I		II		III	

The vſe of this Table, to him that vnderſtandeth the former, is plaine and eaſie. For hauing any breadth giuen, betweene 24 and 1, I ſeeke it amongſt the breadths in that columne vpon the left hand, and in the other columne vpon the right hand oueragainſt it, I find the length deſired.

1 As for example; A boord to be meaſured is 18 inches broade, I deſire to know what length of that breadth ſhall make a foote. ℞ In the ſecond columne, oueragainſt 18, in the firſt columne, I finde 8. Therefore I ſay, euerie 8 inches in length, of that boord that is 18 inches broade, ſhall make a foote of flat meaſure.

2 Againe, ſuppoſe the breadth of a peece of Glaſſe to be meaſured, were 5 inches broad. Here I finde, that euery 28 inches, and $\frac{4}{5}$ of one inch, doth make a foote of that kinde of meaſure.

3 But what if the breadth giuen be greater, or leſſer then any breadth between 24 and 1? I anſwer, the queſtion is anſwered with as great facility. For firſt admit the breadth

giuen were 36, which is greater then any breadth in our Table, here if I take ſome ſuch knowne parte of 36 (*partem aliquotam*, a knowne parte, the Arithmeticians call it) as may be founde in our Table, the length deſired ſhall anſweare vnto it in the like proportion: As for example, if I take 18 the $\frac{1}{2}$ of 36: then it is manifeſt, the length deſired ſhal be but 4 inches, the one halfe of 8, which is the length anſwering to the breadth 18. Or, if I ſhall amongſt the Breadths take 12, which is but the third part of 36, my breadth giuen: then alſo ſhall 4, which is the $\frac{1}{3}$ of 12, the length found, be the length deſired, anſwerable to 36 the breadth giuen: Or which is all one, euery 12 inches in length, of that boord that is 36 inches broade, ſhall containe three foote of flat meaſure.

4 Againe, ſuppoſe the breadth giuen were an 100 inches. ℞. Here 20, is but the $\frac{1}{5}$ part of 100. Therefore I ſay that $7\frac{1}{5}$ in length, of that plaine that is 100 inches broad, ſhall containe fiue foote of flat meaſure.

5 Laſtly, Admitte that the breadth giuen were but $\frac{1}{2}$ inch, which is leſſer then any one of thoſe ſet downe in our Table. ℞, Here I double, treble, or quadruple the number giuen, vntill I may finde it in the Table, and then it is manifeſt, that the length found ſhall be anſwearable to the breadth giuen in the like proportion. As for example, If I double $\frac{1}{2}$, I make 1. Now to the breadth of 1 inch, in the Table there anſweareth 144 inches of length: Therefore the double of 144, that is, 288 ſhalbe the length deſired anſwearing to $\frac{1}{2}$ inch of breadth. Or, which is all one, 144 inches of length of that breadth, that is but $\frac{1}{2}$ inch broade, ſhalbe but $\frac{1}{2}$ foote of flat meaſure. The reaſon of this is manifeſt, out of the 14 element of the tenth booke of Ramus his Geometry.

By this Table you may make a Ruler or inſtrument, wherby whereby the ſame may be performed yet with greater ſpeed and more facilitie. For this inſtrument is none other then that which Artificers do vſe in this caſe. It is, I ſay, no other but a Rule, as they call it, of two foote long (more or leſſe, as you ſhall thinke moſt conuenient for your vſe) and of what breadth you pleaſe, diuided, as the maner is, into inches and partes of inches, with a thwarting or beuelling line drawne from ſide to ſide, bearing the ſeuerall lengths aboue mentioned in our Table. That, for thy better vnderſtanding, we do more particularly thus deſcribe. Firſt with the gage I ſtrike two parallell lines, the one as neare vnto the one edge, the other vnto the other, as ſhall be thought moſt conuenient: Theſe are the breadths, 24 and *6*. Now to the breadth 24, in the Table, there anſwereth *6* inches for the length: Therefore in the vpper line beginning at the right-hand, I account toward the left, *6* inches: There I make a marke. Again, to the breadth *6*, in the ſame Table, there anſwereth 24 inches for the length: Therefore in the neather line counting as afore, I make a mark at the 24 inch. From theſe two markes I draw a line ouerthwart the Rular. In this beuelling or thwarting line, by the helpe of a ſquire, I note all the lengths noted in my Table, betweene 24 and *6*. As for example, To the breadth 23, in the Table, there anſwereth for the length *6* inches, and $\frac{6}{23}$ partes of one inch. This length, I note, beginning mine accoũt as before, in one of the parallel lines. Then by the help of the ſquire I note the ſame vpõ the beuelling line. In like maner I ſet all the reſt of the breadths from this, vnto *6*. This being done the Rular is perfected, and fit for vſe in all caſes, as before is preſcribed in the vſe of the Table.

The *vſe of this Rular* is plaine and eaſie, if either that of the table, or the maner of making this be wel vnderſtood.

For, *If you ſhall ſeeke out the breadth giuen in the beuelling line, the inches & parts of inches from thence vnto the end of the Rular toward the right-hand, ſhall be the length deſired.* I. As for example, Suppoſe the breadth giuen were 12 inches: Here I finde the breadth 12 noted at the twelfth inch from the beginning of the Rular: Therefore I ſay, Euery 12 inches in length, of that plaine that is 12 inches broad, ſhall be a foote of flat meaſure. II. Againe, Admitte the Breadth were *9* inches. This is noted in the thwarting line at *16* inches from the ſayd end. Therefore I ſay, That *16* in length of this breadth, ſhall make a foote of flat meaſure. III. Admit the breadth giuen were *64* inches. Here becauſe I haue none ſo great, I take *16*, the $\frac{1}{4}$ of it. Now the breadth *16*, is noted on the beuelling line, *9* inches from the ſayd end. Therefore I ſay that euery *9* inches in length, of that plaine that is *64* inches broad, doth contain 4 foote of flat meaſure. IV. If the breadth giuen be leſſer then any vpon the beuelling line, then double, treble, or quadruple, &c. the ſame, as afore is taught; and the length found ſhall be but the halfe, third part, or fourth part &c. of the lengh deſired. As for example, Admitte the breadth giuen were 2 inches: Here becauſe vpon the beuelling line I finde no number leſſe then *6*: Therefore I treble 2, and I make *6*. Now *6* being placed vpon the line of breadths, or beuelling line, 24 inches from the ſaid fore-end, I ſay, That 24 inches of length, of that plaine that is but 2 inches broad, is but $\frac{1}{3}$ part of a foote of flat meaſure. V. If the breadth giuen, beſides the whole inches, do containe alſo ſome part or parts of an inch, then that is to be proportioned out between the whole number giuen, and the next greater aboue it. As for example, Admit the breadth giuen were *9* inches & $\frac{1}{2}$: Here the length deſired will fall out proportionally between *9* inches of breadth and 10, aboute 15 inches and $\frac{3}{10}$, from the ſayd end.

Of the meaſuring of Cloth, Wainſcot, Painting, Pauing &c. by the yard.

2 By the Yard we do meaſure Cloth, Wainſcot, Painting, Pauing, &c. The yard, as is aforeſaid, doth containe in length 3 foote, or 36 inches. Therfore the yard-ſquare, or a yard of flat meaſure doth containe 9 foote, or 1296 inches. This kinde of meaſure they commonly diuide into 4 quarters: And euery quarter into 4 nayles.

Vpon the former grounds the like Table and inſtrument for this kinde of meaſure, may be framed as was for that of boord, if any man ſhall thinke it worth the while. But it ſhall not be much loſſe of time or labor, to ſhew in a word or two how it may be done. In a yard-ſquare there is 16 quarters of a yard; or 64 nayles: Therfore if you ſhall diuide 16, or 64, by the breadth giuen, the quotient ſhall be the length deſired. The Table then for this kinde of meaſure is thus:

The breadth.			Length.		
Y.	q.	N.	Y.	q.	N.
0	1	0	4	0	0
0	2	0	2	0	0
0	3	0	1	1	$1\frac{1}{3}$
1	0	0	1	0	0
1	1	0	0	3	$\frac{1}{5}$:
1	2	0	0	2	3
1	3	0	0	2	$\frac{1}{5}$:
2	0	0	0	2	0

The vſe is plaine. 1. Admit a peece of Wainſcotte were three quarters of a yard broade, what length of that breadth ſhall make a yard of flatte meaſure? R. one yard, one quarter, one naile and halfe, or thereabout. The proportion, as afore, is thus: As 4 is to 3, ſo is $5\frac{1}{3}$ vnto 4.

2 A peece of painting is 2 quarters, that is, halfe a yard broade; what length ſhall make a yard ſquare? R. two yards in length.

3 But the vſe of this is better ſeene in buying and ſelling of cloth: For here oft times the skilfull Taylour, although he do well know how much cloth or ſtuffe of this or that breadth wil ſerue to make ſuch a garment, is cleane to ſeeke how much ſhall make the like garment of ſtuffe of any other kinde of breadth: Onely this aduantage he hath, that he wil be ſure to aske enough, which ſhall be no loſſe to himſelfe.

1 Admit 4 yards of cloth of 1 yard broade did make a garment: how much ſtuffe of ¾ broade ſhall make the like? R. The Table for the breadth 1 y, giueth 1 y, for the length of 1 yard ſquare: Therfore 4 yards of this breadth, do make 4 yards ſquare. Secondarily, the ſame Table for ¾ breadth, giueth the length 1 y 1 q 1⅓ n. Therefore foure times 1 y 1 q 1⅓ n, that is, 5 y 1 q 1 n ſhall make the ſame garment.

2 Againe, 12 yards of ſtuffe of ¾ broade, did make a gowne; how much cloth of 1 y 3 q broade ſhall make the like? R. The Table for ¾ breath, doth giue the length 1 y 1 q 1⅓ n: This length I ſeeke how many times I may finde in 12 yards: that is, I ſeek how many ſquare yards 12 yards of ¾ broad doth containe. Againe, the ſame Table for 1 y 3 q breadth, giueth 2 q ½ n. This length ſo many times taken as you found the former length in 12, is the number of yards required.

Of meaſuring of Land by the Aker.

3 BY the Rodde we meaſure Land, Medowes, Wood, Graſſe, Corne, &c. The Rodde, Lugge, Perch, or Pole doth contain, as we haue heard, 16½ foot, or 5½ yards. This kind of meaſure did vary according to their cuſtoms with the countrey: But now by Act of Parliament that

varietie is taken away. The words of which Statute, entituled, *An Act for reſtraint of new buildings, &c. in and neare to the cities of London and Weſtminſter*, made in the xxv yeare of the reigne of Queene Elizabeth, are theſe: Be it enacted by the authoritie aforeſaid, that a Mile ſhall be taken and reckoned in this manner, and no otherwiſe: that is to ſay, a Mile to containe eight Furlongs; and euery Furlong to containe fortie Lugges or Poles; and euery Lugge or Pole to containe ſixteene foote and an halfe. Thus farre that Statute. Now an Aker of land, as we haue ſhewed before, is a plot of ground containing 40 Roddes in length, and 4 in breadth; or, which is all one, an Aker of land containeth 160 ſquare Roddes or Poles of ground. *If therefore you ſhall deuide* 160 *by all the ſeuerall breadths betweene* 40 *and* 1, *the products ſhall ſhew the lengths deſired, anſwering to thoſe ſeuerall breadths giuen.*

The Table then of Land-meaſure is thus:

L	B	L	B	L	B	L	B	L	B	L	B
40	4	20	8	10	16	29	$5\frac{15}{29}$				
39	$4\frac{4}{39}$					28	$5\frac{5}{7}$	14	$11\frac{3}{7}$	7	$22\frac{6}{7}$
38	$4\frac{4}{19}$	19	$8\frac{8}{19}$			27	$5\frac{25}{27}$				
37	$4\frac{12}{37}$					26	$6\frac{2}{13}$	13	$12\frac{4}{13}$		
36	$4\frac{4}{9}$	18	$8\frac{8}{9}$	9	$17\frac{7}{9}$	25	$6\frac{2}{5}$				
35	$4\frac{4}{7}$					24	$6\frac{2}{3}$	12	$13\frac{1}{3}$	6	$26\frac{2}{3}$
34	$4\frac{12}{17}$	17	$9\frac{7}{17}$			23	$6\frac{22}{23}$				
33	$4\frac{28}{33}$					22	$7\frac{3}{11}$	11	$14\frac{6}{11}$		
32	5	16	10	8	20	21	$7\frac{13}{21}$				
31	$5\frac{5}{31}$					20	8	10	16	5	32
30	$5\frac{1}{3}$	15	$10\frac{2}{3}$			19	$8\frac{8}{19}$				

The vſe of this Table is in all reſpects like vnto that of boord-meaſure: Therefore one or two examples ſhalbe ſufficient to make it knowne vnto the ſimpleſt.

I Suppoſe a piece of ground to be meaſured were 32 pole in Length: I demand how much in breadth is required to make an Aker. R. To 32, the length in the firſt columne anſwereth in the next columne 5: Therefore I ſay, euery 5 rods in breadth, ſhall be equall to that field or plot which is 40 pole long, and 4 pole broad.

II Admitte the field to be meaſured were 23 pole long. R. To 23 in the firſt columne I finde anſwering in the ſecond columne $6\frac{22}{23}$, that is, 6 pole, and 22 parts of a pole diuided into 23 parts. Therefore I ſay euery $6\frac{22}{23}$ poles, or 7 pole *ferè*, is equall vnto that which is 40 long, and 4 broad, that is, to 160 ſquare poles.

III But what if the length be greater then any number found in the firſt columne? R. Here as before, I take ſome *partē aliquotā*, as one halfe, one third, one fourth &c. As for example, ſuppoſe the field to be meaſured were 100 poles in length, here becauſe I finde no number ſo great as 100, therefore I take 20, the fifth part of 100: To which 20 I finde 8 pole for the breadth. Wherefore the fifth part of 8 roddes, that is, 1 rodde and $\frac{3}{5}$ partes of one rodde, of that length ſhall be equall to that plot which is 40 long, and 4 pole broade. Or, which is the ſame in effect, Euery 8 pole in breadth, of that field which is 100 pole in length ſhall containe 5 Akers of land.

IV Admit it were 84 pole in length. R. Here the fourth part is 21. Now to 21 pole of length, there do anſwer 7 poles and $\frac{13}{21}$ parts of one pole in breadth. Therefore euery 7 poles, and about two third parts of a pole in breadth, of that length, ſhall make 4 Akers. For if it were but 21 pole long, then euery 7 pole and $\frac{13}{21}$ of breadth, ſhall make one Aker. But 84 containeth the length 21, foure times: Therefore $7\frac{11}{17}$ of the whole length 84, ſhall containe 4 Akers.

Hauing finiſhed this Treatiſe, and it being altogether ready for the Preſſe, turning ouer the Statutes, to ſee what I might finde more for this our purpoſe, I light vpon the very like Table, made many yeares ſince, by Act of Parliament, in the time, as I take it, of Edward the firſt, intituled *De terris menſurandis*. See the latter Abridgement of the Statutes of Raſtall or Poulton, in the title of *Weights and Meaſures.*

A very ſpeedie and readie way or maner of Diuiſion in this caſe of ſurueying.

4 Surueyours, which are often employed in meaſuring of whole Farmes, Lordſhips and Townes, are at laſt to giue an eſtimate of the groſſe ſumme or number of Akers contained in ſome very great ſumme or number of Roddes, after multiplication of the ſides, and addition of many ſeuerall parts, &c. This vſually they performe and finde by diuiding of the product or ſumme by 160, the number of poles that one Aker doth containe. Now here becauſe the Diuiſor conſiſteth of many ſeuerall or ſingle figures, if eſpecially the product or diuidend be any thing great, the worke, as euery one meanly practiſed in Arithmeticke doth know, muſt needs be long, and peraduenture (except great heed be taken) not without ſome error. And againe becauſe, *Faciliùs & expeditiùs numeramus numeros paruos, quàm magnos*, we do more readily, and with greater facilitie number ſmall numbers, then great: If any man ſhall teach how this Diuiſor may be reduced to a ſmall number, conſiſting of one ſingle figure, he ſhall much ſhorten the worke, and deſerue much thanke of all

Surueyors or others delighted in theſe ſtudies. *Ingenui pudoris eſt*, ſaith Plinie, *fateri per quos profeceris*. I confeſſe the inuention is not mine: but I am the firſt, if I be not deceiued, that hath made it *publici iuris*, common to all. Thus then this matter is to be done. *If you ſhall diuide the number giuen, and likewiſe the quotient now found, continually by* 20 *foure times, the laſt quotient (with the parts, if any do remaine) ſhall ſhew the Akers, and parts of an Aker contained in the number giuen*: or, if you ſhall like better, *ſhall diuide the number giuen, by* 40 *twiſe*. Let the examples be thoſe following.

I. Suppoſe a field being meaſured and caſt vp by multiplication, the product or number of perches were 1280, I would know how many Akers it doth containe. Here note out of an abridgement of diuiſion, becauſe my Diuiſor 20, hath for his laſt figure a cipher or nought: therefore I cut off the laſt figure of my Diuidend, thus 128(0.

20) 128(0	
64	Or thus, 40) 128(0
32	32
16	(8
(8	

R. I anſwer by the laſt quotient of both formes, that that field which containeth 1280 ſquare perches, doth containe 8 Akres.

II. Item, Suppoſe the product or groſſe ſumme of ſome peece of land ſurueyed were 102400 perches.

20) 10240(0	
5120	Or thus, 40) 10240(0
2560	2560
1280	(640
(640	

Here I ſay, that 102400 perches, do make 640 akers of lãd.

Theſe examples may ſuffice in ſuch caſes where no fraction ſhall remaine: but if in any of the workes of diuiſion, any parts ſhall remaine, the value of thoſe parts ſhall be eſteemed by theſe rules following.

1 If the firſt figure (the laſt, ſome peraduenture would call it) of the number giuen be a ſignificant figure, then it is manifeſt, that ouer and aboue certaine akers, &c. there are ſome odde perches.

2 Secondarily, if diuiding by 20, there do remaine ought in the firſt diuiſion, that remainder ſhall be $\frac{1}{4}$ of a Roode or 10 perches: if in the ſecond, $\frac{1}{2}$ Roode or 20 perches: if in the third, $\frac{1}{4}$, of an Aker, or 40 perches: if in the fourth, $\frac{1}{2}$ of an aker, or 80 perches.

3 Laſtly, if diuiding by 40, there ſhall ought remaine at the firſt diuiſion, thoſe parts ſhall be parts of a Roode: if at the ſecond, they ſhall be parts of an aker. The examples following, ſhall make all plaine and eaſie.

20) 93(9
46 $\frac{1}{2}$
23
11 $\frac{1}{2}$
(5 $\frac{1}{2}$

Or thus by 40:

40) 93(9
23 $\frac{1}{4}$
(5 $\frac{3}{4}$

Therefore that field that containeth 939 perches, doth containe 5 Akers, 3 Roodes, and 19 Perches.

11 20) 1359(3
679 $\frac{1}{2}$
339 $\frac{1}{2}$
169 $\frac{1}{2}$
(84 $\frac{1}{2}$

Or thus:

40) 1359(3
339 $\frac{3}{4}$
(84 $\frac{3}{4}$

The number 13593, containeth in land-meaſure, 84 A, 3 R, 33 P. or 84 Akers, 3 Roodes, and 33 Perches. The like may be done in Boord-meaſure, and Timber-meaſure, if any man ſhall finde it of any ſuch vſe, as it is in this kinde.

CHAP. IIII.

Of the Extraction of a Cubicke roote.

1 *A Cube is a right-angled ſolid, comprehended of equall ſides: 25 d xj E.*

Hitherto we haue ſpoken of the Quadrate, and of ſuch proprieties and corollaries as did belong vnto this our purpoſe. Now it remaineth that we in like maner do handle the Cube and Cubicke number, and that with as few words, and as briefly as we may. For as the vſes of the Quadrate, and of the extraction of his Roote were many: So the Cube, being of a more excellent nature, cannot yeeld fewer, if not more, and thoſe alſo of more and greater worth. For as Vitruuius, in the preface to his fift book of Architecture, writeth, Pythagoras was ſo much delighted with the Cube, that he wrote all his precepts in cubicall numbers. His words are theſe: *Etiamq; Pythagoræ, hiſq; qui eius hæreſim fuerunt ſecuti, placuit Cubicis rationibus præcepta in voluminibus ſcribere, conſtitueruntq; Cubum 216 verſuum, eoſq; non plus quàm tres in vna conſcriptione oportere eſſe putarunt, &c.* Moreouer alſo Pythagoras, and thoſe which followed his faction, were much delighted to write their precepts and rules of Philoſophie in a kinde of cubicall proportion, making a Cube of 216 verſes; deeming that there ought not to be aboue three in one ſtaffe. Now a Cube is a ſquare bodie, conſiſting of ſixe ſides, plaine and equall one to another. This kinde of bodie, when it is caſt at an aduenture out of the hand, vpon which ſide ſoeuer it pitcheth, no man touching it, ſtandeth firme and conſtant. Such are the dice, which gameſters that play at Tables do vſe. The Greeke comical comicall Poets alſo, which in the midſt of their playes do cauſe the Quiriſters to ſing a ſong, haue ſo diuided the pauſes of their comedies, that making the parts in a kinde of Cubicall proportion, they much, by ſuch reſt, do eaſe and helpe the pronunciation of their actors. Thus farre Vitruuius. This Cube or Cubicall number of 216, hath for his ſide 6. For 6 times 6, do make 36, for the one ſide, or ſquare plaine including the Cube. And 6 times 36, are 216, the Cubicke number of the Cube here mentioned. The myſterie hereof conceiued by Pythagoras and his ſchollers, I leaue to others to vnfold. That pertaineth not to the Mathematician: that other alſo belongeth to the Poets, whereof our age doth affoord plentie. That which the ſame author in another place hath, of the anſwer of the oracle of Apollo: Item, that of Eratoſthenes vnto Ptolomey king of Egypt, of Glaucus his tombe, do more concerne our buſineſſe: And therefore hereafter, in their place, we ſhall handle them at full, if God permit.

Here you ſee how the definition of the Cube doth anſwer to the definition of the Quadrate: for each of them is a right-angled and ſtraight-bounded figure: And as the general differences of bodies or ſolids, were drawne from the generall differences of ſurfaces, plaine and oblique: So here the particular differences are taken from the ſpeciall differences of the ſame ſurfaces. A plaine ſolid is that which is comprehended of plaine ſurfaces: this is generall of what kinde ſoeuer thoſe plaines are of: but a Cube, or cubicall bodie, is that which is comprehended of ſquare or quadrate ſurfaces. The nature then of thoſe ſurfaces which do comprehend them, is that they muſt be plaine, not oblique or vneuen: Secondarily, they muſt be ſquares or *Quadrata*, that is, Right-angled, and of equall ſides. This is Euclids definition, and meaning. The number of

theſe

theſe ſurfaces Vitruuius doth tell: *Cubus eſt corpus ex ſex lateribus æquali latitudine planitierum quadratum*: that is, A Cube is a ſquare bodie comprehended of ſixe plaine ſides of equall breadth. Item, Martianus Capella in theſe words: *Solida figura Quadrati ſex ſuperficies habet*: The ſolid ſquare (ſo he calleth the Cube) hath ſix ſurfaces. From hence is it that they attribute ſtabilitie or conſtancie vnto this kinde of bodie. For *τετράγωνον σῶμα, corpus quadratum*, is ſuch a bodie, if I miſtake not, that conſiſteth of ſuch an equall temperature of the humors, (*eucraſia*,) that it is not ſubiect to that alteration and change that others are. And Ariſtotle calleth a good man *τετράγωνον ἄνδρα, virum quadratum*, that is, *Cubicum*, as I vnderſtand him, meaning ſuch a man as is conſtant, and not eaſily moued vpon euery chance and misfortune, that in this world doth happen to mortall men. Such an one as they report Bias, one of the ſeuen wiſe men of Greece to haue bene. For *Cubus cum eſt iactus*, ſaith Vitruuius, *quam in partem incubuit, dum eſt intactus, immotam habet ſtabilitatem: vti ſunt etiam teſſeræ, quas in alueo ludentes iaciunt*. A Cube when it is caſt out of the hand at an aduenture, looke vpon what ſide ſoeuer it lighteth, it ſtandeth faſt and ſtable; like as thoſe dice alſo do, which gameſters vſe that play at Tables. Therefore, *If ſixe equall ſquares be ioyned with ſolid corners, they ſhall comprehend a Cube.* Item, the ſides of a Cube (*hedra*) are ſixe: the edges (*latera*) are twelue: the plaine-angles, twentie foure: the ſolid-angles are eight; as the author of the Scholium vpon the xv booke of Euclids elements, hath taught.

2 *The power of the Diagonall line of the Cube, is thriſe ſo much as the power of the ſide of the Square comprehending it.*

Harmonia, ſaith our Author, *eſt muſica literatura obſcura & difficilis; maximè quidem quibus Græcæ literæ non ſunt notæ, quam ſi volumus explicare-neceſſe eſt etiam Græcis verbis vti, quod nonnulla eorum Latinas non habent adpellationes*: that is, The theory of Muſicke is very hard and difficult: eſpecially to thoſe which are ignorant of the Greeke toongue: For whoſoeuer ſhall take vpon him to write of this argument, in what language ſoeuer, he ſhall be forced to vſe many Greeke words, becauſe many of them haue no termes anſwerable to them in other languages. The ſame may I ſay of Geometrie, where the moſt termes or words of art are meerly Greeke, or at leaſtwiſe fained in imitation of them, and that oft times not very fitly. For ſurely I doubt not but at the firſt thoſe words ſeemed harſh enough, euen to the firſt inuentors of them: but time and vſe haue made them familiar and pleaſing. Of this ſort is δύναμις, and δύνασθαι, *Potentia*, power, we call it: and *poſſe*, to be able, to be of value and power. Both which do ſignifie in this place or this art, nought elſe but a Geometricall multiplication, as they are oft vſed by Euclid the great Geometer, and Diophantus the ancient Algebraiſt, and that by the teſtimonie of Ariſtotle the Prince of Philoſophers. For of this rule, *Potentia recta eſt quadratum*; or as Diophantus ſpeaketh, *Appellatur quadratus Facultas*, that is, the power of a right-line is a ſquare: the meaning is, that if a right line be multipled in it ſelfe geometrically, it ſhall make a Quadratum or ſquare ſurface. Likewiſe then in this place this propoſition, *Diagonius Cubi poteſt triplum lateris*, The power of a Diagonall line of a Cube, is thriſe ſo much as is the power of the ſide, is thus to be vnderſtood: If the Diagonall of a Cube be multiplied by it ſelfe geometrically; and the ſide of that ſquare which comprehendeth the ſame Cube, be alſo multiplied by it ſelfe, the ſquare that is made of the Diagonall line, ſhall be three times ſo great as is that which is made of the quadrate including the ſame Cube.

Item, againe here obſerue, that that right line which croſſing a plaine from ſide to ſide by the center, is called the Diameter: in a ſolid, the Axis or axeltree; in both, if it paſſe from corner to corner, is properly termed the Diagonius, or the Diagonall line. The power of this line, ſaith our author, is thriſe ſo great as is the power of the ſaid ſide or edge, as I call it. That we thus demonſtrate:

The power of the Diagonius of a Cube, is three times ſo much as the power of the ſide.

That, whoſe power is as much as is the power of any thing ſingle, and of that whoſe power is doubled to the power of the ſame ſingle, ioyntly: is of treble power unto the ſingle. Axio. Logicum.

But the power of the Diagonall line in a Cube, is as much as is the power of the ſingle ſide of the ſquare: and the Diagonall of the ſame ſquare, which is by the 3 & 5 e xij R, double to the ſide.

The power of that ſide in a right-angled triangle, which is oppoſite to the right-angle, is equall to the ſingle powers of the two other ſides. 5 e xij R.

But the Diagonall in the Cube, the ſide of the ſquare, with the Diagonal of the ſame ſquare, do make a right-angled triangle; and this Diagonall of the Cube, is oppoſite to the right-angle, ex theſi.

Therefore the power of the Diagonall of the Cube, is as much as is the power of the ſide, and diagonall of the ſquare.

Therefore the power of the Diagonall line of the Cube, is thriſe ſo much as is the power of the ſide of the ſquare. q.e.d.

3 *If of foure right-lines continually proportional, the Firſt be the halfe of the Fourth; the Cube of the firſt, ſhalbe the halfe of the Cube of the ſecōd.*

This propoſition is a conſequent or corrollary drawne out of the 15 e iiij R, which teacheth that, if certaine right-lines be proportionall (to wit, more in number by one then are the dimenſions of the like figures, alike ſituated vnto the Firſt and Second) it ſhal be as the Firſt right-line is vnto the Laſt, ſo the Firſt figure ſhall be vnto the Second: and contrariwiſe. Now a Cube, by the 1 e xxj R, is a figure of three dimenſions; becauſe it is a bodie, which hath length, breadth and thickneſſe. Therefore here the lines compared in the proportion muſt be foure. The truth of this rule will eaſily appeare by an example in numbers. But here obſerue, that you muſt not expect that our example ſhall altogether directly anſwer to this rule of doubling the Cube; but to ſome other. For truth it is, that Arithmeticke, as hereafter ſhall more plainly appeare, cannot double the Cube; that is, Arithmeticke although it can tell what ſhall be the double of the Cube giuen, yet it can by no meanes tell thee in numbers, what the *Latus* or ſide of that Cube ſhall be. Let the foure numbers continually proportionall giuen, be 2, 4, 8, 16: that is, as 2 is vnto 4, ſo let 4 be to 8: and as 4 is to 8, ſo let 8 be to 16. Here I ſay by that 15 e iiij R, as 2 the firſt number is vnto 16 the laſt: ſo ſhall the Cube of 2 the firſt number, be vnto the Cube of 4 the ſecond. But 2 is but the eighth part of 16: Therefore the Cube of 2 ſhall be but the eighth part of the Cube of 4.

2	4	The Cube of 2,	4	16	The Cube of 4,
2	2	is 8.	4	4	is 64.
4	8		16	64	

Now 2) 16 (8. and 8) 64 (8
16 64

Here you ſee, that the quotient of 16, the laſt number, by 2, the firſt: Item, the quotient of 64, the Cube of the ſecond, by 8, the Cube of the firſt, to be alike, or the ſame: And therefore the proportion is the ſame. q. e. o.

By this rule an anſwer was made vnto that great and ſtrange queſtion, moued by the oracle of Apollo at Delphos. The inuention of this propoſition, that is, of the firſt anſwer to that problem, by ſome is attributed to Plato the diuine Philoſopher, although certaine it is, that Eratoſthenes, in an epiſtle of this argument written to Ptolomey king of Egypt, doth aſcribe it to Hippocrates Chius. The hiſtorie is briefly touched by Vitruuius in theſe words: *Alius enim alia ratione explicare curauit, quod Delo imperauerat reſponſis Apollo, vti aræ eius quantum haberet quadratorum id duplicaretur, & ita fore, vt hi qui eſſent in ea inſula tunc religione liberarentur.* For each of them (meaning Architas Tarentinus, and Eratoſthenes Cyrenæus) labored by ſundry wayes to performe that which Apollo at Delos had giuen in charge in his anſwer, whereby he commanded, that looke how much ſoeuer the altar that was before him ſhould containe in ſquare feete, it ſhould be doubled, and ſo it ſhould come to paſſe, that thoſe in that Iland, which were ſicke of the plague, ſhould be freed from that curſe. But Eratoſthenes before mentioned, layeth downe the hiſtorie more largely and plainly, thus: To king Ptolomæus, Eratoſthenes ſendeth greeting; It is reported that one of the ancient tragedians bringeth in Minos, purpoſing to reedifie Glaucus his tombe. And being giuen to vnderſtand, that it was euery way an hundred foote, ſaid, That is too little for the ſepulcher of ſo great a king. Let it be therefore doubled. It ſeemeth that Minos vnderſtood not what he ſaid: For if you ſhall double the ſides, the plaine ſhall be foure times ſo great, and the ſolid eight times. It was demanded therefore of the Geometricians, How the ſolid giuen, might be made iuſt as big againe as it was at the firſt, and yet the forme and nature of the figure to remaine the ſame ſtill. This propoſition is called, The doubling of the Cube. For making the queſtion of the Cube, they laboured to make that double or as big againe. All men therefore doubting and ſtudying a long time how this ſhould be done, at length Hippocrates Chius found that, *If vnto two right-lines giuen, whereof the greater ſhould be double vnto the leſſer, two meane proportionals might be found, then the Cube might be doubled.* But thus this doubt was to be reſolued by another thing, as difficult and as hard to be done as that other. Within a while after, they report that the citizens of Delos being commanded by the Oracle to double a certaine altar, and falling into the ſame doubt and difficultie, were conſtrained to ſeeke vnto the Geometricians which were with Plato in the Vniuerſitie, that they would at their requeſt, finde out ſome way how to performe the ſame. Now theſe ſtudents diligently conferring vpon the matter, and laying their heads together to finde out how, vnto two right-lines giuen, two meane proportionals might be found, it is ſaid that Architas Tarentinus found them by ſemicylinders, and Eudoxus by thoſe lines that are called crooked lines, *καμπύλας, curua, &c.* Thus farre Eratoſthenes. How then to ſatisfie the demaund of the citizens of Delos, to performe, I ſay, the command of Apollo and Minos, was demonſtrated by Plato, or Hippocrates Chius, to wit, by the finding out of two meane proportionals, betweene two right-lines giuen, whereof the one is double vnto the other. But now againe here ariſeth ther queſtion, or as Eratoſthenes ſaith, τὸ ἀπόρημα αὐτοῦ εἰς ἕτερον οὐκ ἔλαττον ἀπόρημα κατέστρεφον, that this doubt of Apollo was to be reſolued by another doubt, no leſſe difficult then it ſelfe. For how this was to be done, had not as then by any before that time bene taught. But vpon theſe occaſions giuen (moued alſo partly with glorie) many fine wits in ſundrie ages haue found out diuers and ſundrie wayes how to performe the ſame, as Eutocius Askalonita, Archimedes his learned interpreter, hath at large ſet downe in his commentarie vpon his ſecond booke *De cylindro.* Theſe authors, as there you ſhall finde, are thus and in this order named by him: Plato, Heron, Philo Bizantius, Apollonius, Diocles, Pappus, Sporus, Menechmus, Architas Tarentinus, Eratoſthenes, Nicomedes, and Eudoxus Cnidius, whoſe inuention he reiecteth, as not anſwering to the purpoſe of his owne propoſition. Of all theſe P. Ramus, at the 8 e xiij of his Geometrie, deſcribeth that of Heron onely, neglecting all the reſt. *Hinc exiſtit*, ſaith he, *Meſographus Heronis Mechanici ſeu Meſolabus, dictus ab inuentione duarum continuè mediarum proportionalium inter duas datas: vnde exiſtit problema Deliacum quod Apollinem ipſum exercuit. Meſographus autem Heronis eſt infinita regula, quæ ſiſtitur cochleato vnco per cauum mobili. Eſt verò, vt Pappus ait initio libri tertij, Meſographus iſte architectis aptiſsimus, multoq́; promptior Platonis Meſographo. Meſographi mechanica eſt apud Eutocium ſecundo de ſphæra, ſed paulò faciliùs à nobis ita proponetur: Si duas datas rectas comprehendentes rectangulum, & infinitè continuatas, meſographus tangens oppoſitum angulum angulo datarum interſecet æquidiſtantèr à centro, interſegmenta erunt media continuè proportionalia datis:* that is, From hence (he meaneth out of the 7e of that booke) ariſeth the Meſographus of Heron the inginer, which otherwiſe is called a Meſolabus: For the vſe of it is to finde out two lines continually proportionall betweene two other lines giuen. By this meanes was found how to anſwer that problem of Delos, which did much trouble Apollo himſelfe. Now this Meſographus of Heron, is an infinite right-line which is faſtened with a ſcrew-pin, that is to be ſlid vp and downe in a riddle. This Meſographus, as Pappus Alexandrinus in the beginning of his third booke writeth, is a maruellous commodious inſtrument for Architects and Carpenters, and is much more conuenient and readie then the Meſographus of Plato. A mechanicall deſcription and vſe of this Meſographus, is ſet downe by Eutocius in his commentarie vpon the ij booke of Archimedes *De ſphæra & cylindro*, but is ſomewhat more plainly ſet out by vs in this maner: 8 *If (a right-angled parallelogramme being made of the two lines giuen, the ſame alſo being continued as farre as need ſhall require) the Meſographus touching the angle, that is oppoſite vnto the angle contained of the lines giuen, ſhall cut thoſe continued lines equally diſtant from the center, the portions of thoſe continuations thus cut, ſhall be the middle lines continually proportionall betweene the two lines giuen.* Thus farre Ramus, vnto whom I referre thee for further ſatisfaction. But whereas P. Ramus, as we haue ſhewed before, doth affirme that the Meſographus of Heron, as Pappus Alexandrinus writeth in the beginning of his third book, is a maruellous cōmodious inſtrument for architects, and is much more readie and conuenient then the Meſographus of Plato; it is ſo to be vnderſtood, that thoſe words, *Multoq́; promptior Platonis meſographo*, be not ſuppoſed to be ſpoken by Pappus: For indeed in that place he doth not once name Platoes meſographus. His words at the 4 p iij, as Commandinus, his interpreter hath expreſſed them, are theſe: *Exponemus igitur quatuor eius conſtructiones, vnà cum quadam noſtra tractatione; quarum prima quidem eſt Eratoſthenis, ſecunda Nicomedis, tertia Heronis maximè ad manuum operationem accommodata, ijs qui architecti eſſe volunt.* Of thoſe many and ſundry inſtruments, that haue bene found out by diuers men for the inuention of two proportionals, &c. we will deſcribe the vſe and making of foure, adding thereto a certaine treatiſe of our owne. The firſt is Eratoſthenes his way, the ſecond is Nicomedes, the third is Herons, moſt commodious for all handy-crafts, or mechanicks, and ſuch as are deſirous to be architects. Here you ſee is no mention of Platoes inuention. But howſoeuer, that of Platoes, in mine opinion, is more proper to this place and purpoſe of ours; becauſe it is to be done with the Carpenters tooles, which are alwayes at his hand: And therefore we wil here alſo, out of the ſame Eutocius, deſcribe that more fully. This Meſographus, or inſtrument deuiſed by Plato, is deſcribed by Eutocius to haue bene a right-angled parallelogramme, conſiſting of foure ſtraight rulars, ſo contriued and put together, that they might be put nearer or farther off, as occaſion ſhould require. We haue vſed two Carpenters ſquires, ſo placing the ſide of the one ſquire vpon the ſide of the other, that the other two ſides might continually be parallell, and containe right-angles betweene them. Or if any man ſo pleaſe, one ſquire may ſerue the turne with the rule which they vſe in meaſuring. The inſtrument now according to theſe directions, is thus to be vſed: *If* (the two lines giuen comprehending a right-angle, and from the angle infinitely continued) *the inſtrument be ſo applied, that when in each corner the infinite lines do fall, the ſides of the ſame inſtrument do touch the ends* *of the lines giuen; the portions of the continued lines intercepted between the ſaid corner, and the corners of the inſtrument, ſhall be the two middle lines continually proportionall between the two lines giuen.* Concerning the Diagramme, I muſt referre thee either to Eutocius vpon Archimedes, or Daniel Barbarus vpon Vitruuius: Let the inſtrument be f m, the ſquire: The ruler to be moued vp and downe, let it be n o. The vſe according to our rule is thus: Let the two lines, whereof the one is double to the other, be e b, and b g; the one falling vpon the other perpendicularly, that is, they both containing the right-angle e b g. Again, let e b, be drawne out infinitely toward c: and g b, in like maner toward d. Now apply the inſtrument to this figure in this maner, that when by mouing n o, the moueable ſide, vp and downe, the two continued lines falling preciſely in the corners m, n; the two parallels ſides m, and n o, may at the ſame inſtant touch e and g, the ends of the lines giuen. I ſay that b d, and b e, the portions of the continued lines, cut or intercepted betweene the corner b, and n, m, the corners of the inſtrument, are the two middle lines continually proportionall betweene e b, and b g, the two lines giuen. The words of Daniel Barbarus at the 3 chap. of the ix booke, are theſe: *Coniungantur in b, ad rectum angulum duæ rectæ, inter quas duæ comparabiles, ac ratione pendentes medias vis inuenire. Eſto b g, maior: e b, minor: vtraq; verò extra angulum b producatur: Maior, ad d: minor, ad c: & ad duos rectos angulos, vnum in c; alterum in d: in ſuis reſpondentibus lineis. Eſtoq; angulus vnus, g c d; alter c d e: Aio inter duas datas e b, & b g; eſſe duas medias comparabiles inuentas: bd, ſcilicet, & b c. Quoniam poſuimus angulum e d c rectum: & c d aquediſtantem ipſi e g; Ideo ſequitur, ex 29 e j, angulum g c d, rectum eſſe: & aqualem angulo c d e, quem ſimiliter rectum eſſe poſuimus. Sed a b, ex conſtructione ſupra* *c b e, ad rectos cadit, ſimiliter c b perpendicularis eſt ipſi d b g. Ex corollario itaque* 8 e vj, *b d eſt comparabilis illi qua cadit inter c b, & b c: Pari quoque ratione b c, eſt media inter b d, & b g. &c.* Vide pag. 276 & 277.

That d b, and b e, are continually proportionall to c b, and b g.

Continua proportio eſt, in qua idem terminus pro ſecundo & tertio ſumitur. *A continuall proportion is that where the ſame bound, is taken for the ſecond and third:* 8 *e* 2 *c ij Salig.*

But here d b: and c b, the ſeuerall meane bounds, are each of them taken for two:

1°

A plum-line falling from a right-angle vpon the baſe, is the meane proportional betweene the portions ſo made: 1 *c* 4 *e viij R.*

But e d c, being a right-angle (ex theſi & fabrica) *and d b falling plum vpon e c, cutteth the portions e b, and b c.*

Therefore d b is the meane proportionall betweene e b, and b c: that is, as e b, is to db, ſo is bd, to bc.

2°

A plum-line falling, &c. 1 *c* 4 *e viij R.*

But d c g being a right-angle; and c b falling vpon d g, doth make the portions d b, and b g.

Therefore c b, is the meane proportionall betweene d b, and b g: that is, as d b, is to b e: ſo is b e, vnto b g.

Therefore d b, and b e, are continually proportionall betweene c b, and b g, the two lines giuen. That is, as c b, is to d b; ſo is d b, vnto b e: And as b d, is vnto b c; ſo is b e, vnto b g. quod erat demonſtrandum.

Thus you ſee, that that which was then ſo difficult, how eaſily it is now to be done, and that by many and ſundrie maner of waies; and yet neither of theſe do ſatisfie the Geometrician: The reaſon is, for that, as Pappus Alexandrinus ſaith, thoſe authors *quod natura ſolidum eſt, Geometrica ratione innixi, conſtruere non potuerunt. Inſtrumentis enim tantùm ipſum in operationem manualem, & commodam, aptamq; conſtructionem mirabilitèr traduxerunt.* Becauſe, ſaith he, thoſe authors, although they haue all ſhewed how to do this wonderfully fitly and well; yet all of them haue done it but mechanically onely, by the helpe of certaine inſtruments, none of them yet hauing found how it may be done geometrically.

4 *The ſolid number of a Cube, is called a Cubicke.*

5 *A Cubicke number is that which is made by multiplying of any number giuen into his quadrate.*

A Quadrate number is a number that is made by the multiplication of a number in it ſelfe. Therfore a Cube is a number which is made by the multiplication of three equall numbers; or, a Cubicke number is that which is made by the multiplication of any number by it ſelfe, and againe, by the multiplication of himſelfe into the ſaid product. As for example, 2 by 2, do make the quadrate 4: Now 2 by the ſaid 4, do make 8, a Cubicke. Of this number, Martianus Capella thus writeth: *Octonarius numerus, primus cubus eſt, & perfectus, Vulcano dicatus: Nam ex primo motu, id eſt, Diade, quæ Iuno eſt, conſtat. Nam dias per diadem, facit tetradem: At bis ducta, facit octadem. Perfectus item, quòd à ſeptenario tegitur. Omnis enim Cubus ſex ſuperficies habet. Item, Ex imparibus conſecutis impletur. Nam primus imparium trias, ſecundum pentas, ambo octadem faciunt. Item, Cubum, qui à triade venit, id eſt, 27, ſequentes impares reddunt, id eſt, heptas, enneas, & 11: qui omnes faciunt 27. Item, Tertius Cubus, qui à tetrade venit, id eſt, 64. Nam quater quaterni ſunt ſedecim. Hoc quater, 64 fit. Et hic ex imparibus*

bus quatuor, qui ſuperiores ſequuntur, id eſt, 13, 15, 17, 19, *fiũt ſimul* 64. *Et ſic omnes Cubi per imparium incrementa inueniuntur, ſui duntaxat numeri. Sanè hic octonarius Cubus, ita omnium Cuborum primus eſt, vt monas omnium numerorum. Cubus autem omnis etiam matri Deûm tribuitur: Nam ideò Cybele nominatur.* Thus farre Capella. This rule is conceiued in few words thus: *Digeſtis à ternario imparibus, ſi duo priores; poſtea tres; deinde quatuor, &c. coniungantur, Cubos proferunt*: that is, If odde numbers digeſted according to their naturall order, be added, (an vnitie onely, which is of it ſelfe a Cube, excepted) firſt two; then three; then foure, &c. they ſhall make the Cubes naturally following one another, thus:

1	3	5	7	9	11	13	15	17	19	21	23	25	27	29
1		8			27				64					125

[1] Therfore looke of how many vnities the radix or ſide of any Cube doth conſiſt, of ſo many odde numbers is the ſame Cube compoſed and made. Item, [2] If the firſt figure (I meane that on the right hand) of a Cube giuen, be an odde number, the number of odde numbers whereof it was made, is odde: If it be an euen number, the number of odde numbers whereof it conſiſteth, is euen. Item, [3] If a Cube do multiply a Cube, the product ſhall be a Cube; and the ſide of that Cube ſhall be the product made by the multiplication of the ſides of the Cubes giuen. As for example, Let the two Cubes giuen be 8 and 27; whoſe ſides let them be 2 and 3: Now let 8 multiply 27, and let the product be 216: I ſay the ſide of the Cube 216, is 6, which is the product of 2 by 3. Thus haue you an eaſie way, by the helpe of the xiiij Chapter of the ſecond book of Salignacus his Arithmetick, how to finde out the cube of any place, and the ſide thereof, without any farther adoe, onely by the helpe of one multiplication. Now the

ſides of cubes leſſe then 1000, being ſingle figures, to wit, what cube is made of euery ſingle figure multiplied in it ſelfe cubically, and what the ſide of euery ſuch cube is, muſt firſt be knowne before we go any farther: That is done by this table:

1	2	3	4	5	6	7	8	9:	Sides.
1	4	9	16	25	36	49	64	81:	Quadrates.
1	8	27	64	125	216	343	512	729:	Cubes.

6 *If a right-line be cut into two portions, the Cube of the whole line ſhalbe equall to the Cubes of the ſegments, and a double ſolid thriſe comprehended of the quadrate of his portion, and the other portion.*

The generall inuention of a Cube, both Geometricall and Arithmeticall, was ſhewed at the former propoſition: The ſpeciall or particular inuention of the ſame is diuers and manifold by numbers continually proportionall and out of the cubes themſelues, as Euclid teacheth, of whom thou mayſt learne, if thou ſhalt thinke the fruite of that knowledge may counteruaile thy trauell. *Analyſis quadrati lateris*, The extraction of a ſquare roote, as they call it, hath in Euclids Geometrie a proper element, theoreme, or rule, teaching how to performe it; but the extraction of a cubicke roote hath not any at all: Notwithſtanding according to that of the ſquare by him laid downe, it is not difficult by analogie to make one, whereby the roote alſo of the Cube may be found. The propoſition therefore according to that analogie, teaching this skill, is thus layd downe by Ramus: If a right-line be cut into, &c.

As for example, Let the ſide or roote 12, be cut into two portions, 10 and 2: I ſay the Cube 1728, made of 12, the whole line, is equall to 1000, and 8, the cubes made of 10 and 2, the ſaid portions: and two diuers ſolids, whereof the firſt is 600, comprehended thriſe of 100, the ſquare of 10, his ſegment; and of 2, the other ſegment: The ſecond, 120, which is thriſe comprehended of 4, the ſquare of his ſegment 2, and of 10 the other ſegment. But the frame and making of the whole cube, will make the matter more plaine and eaſie in one example, to wit, how the outter and meane ſolids are made. Let therefore a cube be made of 12, 12, and 12, three equall ſides: and firſt let the ſecond ſide be multiplied by the firſt, thus:

12
12
24
12

Let not the products 24, and 120, be added together; but let the other ſide multiply them ſeuerally, and then adde the ſeuerall degrees by themſelues, thus:

24
120
12
48
240
240
1200
128
16
1728

Or thus,

24
12
12
48
24
24
12
8
120
600
1000
1728

This is the making the Cube of the whole line: Now the making of the ſame according to the former diuiſion of the ſaid line, is thus:

I. The cubes of the ſegments, 10 and 2.		II. The ſeuerall ſolids.		III. The ſeuerall parts added.
10	2	10	2	1000
10	2	10	2	8
100	4	00	4	200
10	2	10	10	200
1000	8	100	40	200
		2		40
		200		40
				40
				1728

But in this maner which followeth, of our inuention, the particular ſolids, in this kinde of making the cube of the ſeuerall ſegments, do more plainly appeare, then in that practiſed by Ramus: as theſe examples do ſhew.

I.	II.	
12	22	
12	22	
4	4	The Quadrate of the leſſer ſegment.
20	40	The Oblongs of the greater and leſſer ſegments.
20	40	
100	400	The Quadrate of the greater ſegment.
12	22	
8	8	The Cube of the leſſer ſegment.
40	80	The leſſer ſolids made of the Quadrate of the leſſer ſegment, and of the other ſegment ſingle.
40	80	
40	80	
200	800	The greater ſolids made of the quadrate of the greater ſegment, and of the leſſer ſegment ſingle.
200	800	
200	800	
1000	8000	The Cube of the greater ſegment.
1728	10648	The Cube of the whole line.

Now that we may apply this example vnto our rule, adde the ſolids of the ſame kinde and qualitie together, that is, 40, 40 and 40, together: Item, 200, 200 and 200 together, and adde the ſummes of them, 120, and 600, vnto 1000, and 8, the cubes of the ſegments, and the ſum 1728, ſhall be equall vnto 1728, the cube of the whole line.

40	200	8	The leſſer Cube.
40	200	120	The ſum of the three leſſer ſolids.
40	200	600	The ſum of the three greater ſolids.
120	600	1000	The greater Cube.
		1728	The Cube of the whole line.

If any man ſhall thinke this rule to be true onely in this caſe where the line is diuided according to the nature of the number, thus conſiſting of two digits, as they call them, he is deceiued: For the ſame effect will fall out, howſoeuer the line ſhall be deuided. Let therefore the ſame number firſt be diuided into two other ſegments vnequall betweene themſelues, to wit, into 8 and 4.

I. The cube of the great ſegm.		II. The cube of the leſſer ſegm.		III. The firſt ſolid.			IV. The ſecond ſolid.			
8	64	4	16	8	64	256	4	16	128	64
8	8	4	4	8	4	3	4	8	3	512
64	512	16	64	64	256	768	16	128	384	768
										384
										1728

Secondarily let the ſame line be cut into two equall ſegments, to wit, 6, and 6.

I. The firſt cube.		II. The ſecond cube.		III. The firſt ſolid.			IV. The ſecond ſolid.			
										216
										216
6	36	6	36	6	36	216	6	36	216	648
6	6	6	6	6	6	3	6	6	3	648
36	216	36	216	36	216	648	36	216	648	1728

Therefore, if the right-line giuen be cut into two equall ſegments, the ſeuerall ſolids of the ſegments ſhal be equal vnto the Cubes that are alſo equall betweene themſelues, that is, there ſhall be eight ſolids all equall one vnto another: or, that which is all one, the ſolids are the ſame with the cubes. Therefore

7 *The ſide of the firſt particular Cube, is the one ſide of the ſecond ſolid, and the ſquare of the ſame ſide, is the other ſide of the firſt ſolid, whoſe other ſide is the ſide of the ſcond Cube; and the ſquare of the ſame other ſide, is the other ſide of the ſecond ſolid.*

In this equalling then of foure ſolids with one ſolid, there is to be obſerued a ſingular kinde of frame and compoſition: Firſt, that the laſt cube be made of 2, the laſt ſegment: Then, that the ſecond ſolid made of 4, the ſquare of 2 his ſegment, and of 10, the other ſegment, be thriſe taken. Againe, that the firſt ſolid made of 100, the ſquare of 10 his owne ſegment, and of 2 the other ſegment, be alſo thriſe taken. Laſtly, that the cube 1000, be made of 10, the greater ſegment. Out of this frame or making of a Cube, the contrary *Analyſis* or reſolution of the ſame is deriued, out of the mutuall combination of the Cubes with the ſolids, ſuch as we haue before ſhewed in the *Analyſis* of a ſquare. For here, although a ſolid be named onely, yet there are two ſides to be conſidered: becauſe that the one is compound and plaine. Therefore 10, the ſide of 1000, the firſt cube, is the key for the opening of the two ſolids and cube following. For it is the one ſide of the ſecond ſolid, to wit, thriſe comprehended of it and of the ſquare of the ſecond ſegment. Againe, 100, the ſquare of the ſame ſide 10, is the one ſide of 600, the firſt ſolid, to wit, thriſe comprehended of this quadrate 100, and the other ſegment 2. Item 2, the other ſide of the firſt ſolid, is the ſide of the next cube. And laſtly, the ſquare of 2, the ſame ſide, is the other ſide of the ſecond ſolid. By this meanes then, the great varietie and difficultie of this buſineſſe is vnfolded, like as was done in the ſquare. For when you haue found the ſides of the ſeuerall cubes, then haue you withall alſo found the ſide of the whole cube: For although the whole Cube be greater then the cubes of the parts; yet the whole ſide is equall to the ſides of the ſeuerall cubes: For we vſe the ſolids that are conſidered betweene the cubes, onely as a meanes to finde out the ſide of the following cube. Thus much concerning the true forme of analyſing or reſoluing of a Cube: But becauſe this may ſeeme ſomewhat difficult and hard vnto a learner, out of this propoſition we haue framed another, which doth as it were more diſtinctly expreſſe and point at euery particular in this practiſe.

8 *If the ſide of any greater Cube ſhall be ſought, thou ſhalt from the right-hand toward the left diſtinguiſh the number giuen into perfect periods, for ſo many particular Cubes: Then hauing found the ſide of the firſt particular Cube, thou ſhalt ſet it downe within the quotient: Againe, hauing ſquared this ſide now found, thou ſhalt treble the product for the baſe of the firſt ſolid; but for the height or other ſide of the ſecond ſolid, thou ſhalt onely ſquare it. Laſtly, the firſt ſolid diuided by his baſe, the quotient ſhall be the*

the ſide of the next following Cube.

As for example; Let the ſide of the cube 1728, be ſought. I. Here firſt beginning at the right-hand, I diſtinguiſh it into perfect periods, that is, into three degrees thus: 1,728. Now becauſe after this diſtinction it appeareth, that our number giuen is a compound period, compoſed of two ſingle periods; therefore by the former rule I ſay that the whole conſiſteth of two particular cubes. II. Secondarily, the firſt particular cube being 1, I ſeeke his ſide amongſt the cubes of ſingle figures, at the 5 e, and I finde the ſide to be 1: that I ſet downe therefore in my quotient. Now, as the maner is in diuiſion, I ſet downe 1, the cube of this quotient or ſide, vnderneath the firſt figure of the number giuen: Item, ſubtracting the one out of the other, I cancell all the figures of the number giuen, ſetting the remaine, or the whole next period, aboue the head right ouer their degrees. III. Thirdly, I ſquare 1, the ſide found, and I make 1, which I treble for the baſe of the firſt ſolid, and I make 3: by this baſe, or treble 3, I diuide 7, the firſt ſolid or firſt complement; the quotient 2, I place in the quotient for the ſide of the ſuccedent cube: By this quotient 2, I multiply 3, the Diuiſour, and I make *6* for the firſt ſolid: Then by 4, the ſquare of the ſame quotient, I multiply 3, the treble before reſerued; and I make 12 for the ſecond ſolid: Thirdly, I multiply 2, the quotient cubically, and I make 8: Fourthly, placing all theſe products in their true places, iuſt one degree behinde another, ſo that the firſt ſolid be placed in the firſt degree; the ſecond ſolid one degree farther toward the right-hand: the cube one degree farther then that, or next of all to the right-hand, in this maner:

```
 6
 12
  8
```

Again,they being thus placed,I adde them, and I find the ſumme 728: Now this ſumme I ſubtract from the remaine 728: Laſtly, becauſe after ſubtraction there remaineth nothing, I ſay,that 1728,the number giuen,is a cubicke,and the ſide of it is 12. The example of this practiſe is thus:

728
1728
1
728

But here obſerue, for that the firſt ſolid (or complement) is the third degree from the ſecond cube; and the ſecond ſolid,the ſecond or next vnto the ſame;therefore vnto the firſt ſolid I adde two ſiphers,and vnto the ſecond but one: For theſe figures do ſo guide the practitioner,that he cannot eaſily erre in the addition of them. Or thus, as Salignacus an excellent artiſt,ſetteth it downe. Let the ſide of the cubicke 389017,be found.Here firſt,becauſe that 389 is not found amongſt the cubes at the 5 e, that is,becauſe that 389 is not a cubick number, therefore I take 343,the next leſſer cubicke before it, whoſe ſide 7 I place within the quotient. II. Then taking 343 out of 389,there do remaine 46: Therefore here the two complements with the ſecond cube,are 46017. III.This done,I ſquare 7 the ſide found,and I make it 49. IV. This ſquare I treble, and I make 147,for the baſe of the firſt complement. V. Againe, I treble the ſide found,and I make 21 for the height of the ſecond complement. VI. Laſtly, diuiding 460, the firſt complement by 147 his baſe,I find the quotient 3,which I ſet downe in the quotient, for the ſide of the ſecond cubicke.

For the proofe of this worke or practiſe, Firſt we multiply the quotient in it ſelfe cubically: Secondly,we multiply

tiply the height of the cube by the baſe of the firſt complement: Thirdly, we multiply the baſe of the ſame cube, by the height of the ſecond complement: Fourthly, we adde all theſe products together: Laſtly, we ſubtract the ſumme found, out of the vpper number. As for example: I cube 3, or multiply it in it ſelfe cubically, and I make 27. Now the height of this cube is 3; the baſe is 9: Therefore I multiply 3, this height of the cube, by 147, the baſe of the firſt complement, and I make 441. Againe, I multiply 9, the baſe of the ſaid cube, by 21, the height of the ſecond complement, and I make 189. Now I adde all theſe products, and the ſumme is 46017: which ſumme ſubtracted out of 46017, the vpper number, nothing remaineth. Therefore if you ſhall ſquare 73, the quotient; and then ſhall multiply this ſquare by the ſame 73, thou ſhalt make the cubick 389017. For the cubes of the ſegments 70, and 3, with the two complements now found, are the parts of the cube that is made of the whole 73; as may eaſily be tried by that which we haue taught in the former. Briefly then, and in this order is this whole worke to be done. Let the example be the cube 1728. I. By the 5 e, I finde the ſide of the cubick 1, to be alſo 1, which I ſet downe in the quotient. II. This quotient now found, I treble, and I make 3. III. Againe, this treble I multiply by the ſayd quotient, and I finde 3 alſo for the Diuiſor. By this Diuiſor 3, I diuide 7, the firſt complement or ſolid: and I finde the quotient 2 for the ſide. IV. Now by this quotient I multiply the Diuiſor 3, and I make 6, for the firſt complement: Therefore to it I adde two cyphers, and I make 600. V. Againe, I ſquare the quotient 2, that is, I multiply it by it ſelfe, and I make 4: This 4, I multiply by the treble 3, and I make 12, for the ſecond complement: Therefore I adde to it one cypher, and I make 120. VI. This done, I multiply the ſame quotient 2, in it ſelfe cubically, and I make 8. VII. Laſtly, placing all theſe orderly one vnder another, I adde them, and I finde the ſumme 728, which ſubtracted from 728 the number remaining, nothing is left: Therefore I ſay that 1728 is a cubicke.

9 *If, hauing found the quotient of two or more figures, as yet the whole ſide of the Cubicke giuen be not found, then for the finding out of the quotient following, thou ſhalt ſquare the whole quotient now found for the baſe of the next complement; and ſhalt treble the ſame ſquare; and then ſhalt in all things follow the preſcript of the former rule, the whole ſide ſhall be found.*

Suppoſe the ſide of 34,012,224, were to be ſought: Item of 320,013,504: The examples would be thus:

```
          1244224                      5581504
          7012                         104013
   27) 34,012,224 (324||108) 320,013,504 (684
          27              13872) 216
 3072)    5768                       98432
          1244224                      5581504
```

II. The

	II. The treble of the whole quotient.	III. The product of the treble by the whole side.	IV. The product of the quotient by the Diuiſor.	V. The product of the ſquare of the quotient by the treble.	VI. The cube of the quotient now found.	VII. The ſum of all thoſe three products.
I.	1 3 3	3 1 3	3 2 600	2 3 2 4 4 120	2 4 2 2 4 8	600 120 8 728
			~~211~~8~~4~~ 48) 8~~51~~8~~4~~ (44 6~~4~~ ~~211~~8~~4~~			
II.	4 3 12	12 4 48	48 4 19200	4 16 4 12 16 1920	4 16 4 4 16 64	19200 1920 64 21184
			~~55~~8~~1504~~ ~~10~~4~~013~~ 108~~0~~) ~~32001354~~ (684 13872~~0~~) ~~21~~6 98~~431~~ ~~55~~8~~1504~~			
IV.	6 3 18	18 6 108	108 8 864	8 64 8 18 64 1152	8 64 8 8 64 512	86400 11520 512 98432
	68 3 204	204 68 13872	13872 4 5548800	4 204 4 16 16 32640	4 16 4 4 16 64	5548800 32640 64 5581504

Somtimes after that you haue found the cubicall ſide, I meane, hauing ſubtracted the cube of the firſt period, there will remaine in the next places neither ſolid nor cube. Therefore in this caſe vnto the ſide found, you ſhall adioyne a cipher, as in this cube 8120601 (201.

If the ſecond ſide of the firſt complement be greater then the ſide of the cube following, then the ſolid diuided did containe a part of the ſecond complement: And therfore that ſide muſt be diminiſhed, as in the cube 17,576, the firſt ſide ſhall be 2; then if thou ſhalt make the ſecond ſide 7, as it ſeemeth at the firſt view it ſhould be, thou ſhalt make the ſecond ſide of the firſt complement greater then the ſide of the cube following. Therefore that ſide muſt be taken leſſe, and for 7 I take 6, and ſo thou ſhalt finde the true ſide of the cubicke giuen to be 26. The example is thus:

9576
27,576 (26
8
9576

This is the generall and common way of finding of the ſide of any cubick whatſoeuer, though neuer ſo great. Thus alſo parts, whoſe bounds are cubicall numbers, may be reſolued into leſſer bounds, by finding out of the ſides of the ſame bounds; As $\frac{8}{27}$ by this meanes are reſolued into $\frac{2}{3}$. And $\frac{1728}{3375}$, into $\frac{12}{15}$.

In parts (fractions they commonly call them) there is alſo a kinde of cube. *For if ſo be the number giuen be not a cubick, then it hath no ſide that may be expreſſed in numbers; and yet the true ſide of the greateſt cube contained in any number giuen, may be found*: As in this number 17,616, which is not a cubicke, the greateſt cubicke is 17,576, and the ſide of it is 26, and there do remaine, ouer and aboue the cubicke,40. Therefore, *there cannot poſsibly any ſide of a number that is not a cubicke, be found ſo neare, but it is poſſible to finde one more neare*; as is alſo before taught of the Quadrate. There are alſo here two maner of wayes of finding out a cubicall ſide in ſuch like numbers very néare to the true ſide, ſuch as were before ſhewed in the quadrate. *The firſt ſubducteth the two complements and the laſt cubick, contained in the next greater cubicke aboue it*; whereby may be vnderſtood the difference of two continuall cubickes, like as before the difference of two quadrates was vnderſtood. For here there is to be vnderſtood a certaine cubicall Gnomon or ſquire, which you may conceiue to be made of three plaines or ſides of the cube, as before you conceiued the ſquire or gnomon of the quadrate to be made of two ſides of the quadrate. So in this example 17,616, where the ſide is 26, and there do remaine 40, of the cube next following, thou ſhalt diuide 27, the next greater cubicke, into 26 and 1, and, as before thou haſt learned, thou ſhalt make two complements and one ſolid of them. The firſt complement 676, made of the ſquare of 26, the one ſegment; and of 1 the other ſegment; which being thriſe taken, ſhall be 2028. The ſecond complement is 26, comprehended of the ſquare of 1, the one ſegment; and of 26, the other ſegment: which being thriſe taken, do make 78. Now the cube of 1, is 1. This done, adde all theſe, and the ſumme 2107, ſhall be the denominator of the fraction ſought. The parts therefore to be added vnto 26, the former ſide, ſhall be $\frac{40}{2107}$. So that the whole ſide of 17,616, the number giuen, ſomewhat neare vnto the true ſide, is 26$\frac{40}{2107}$. Wherby it is to be vnderſtood as aboue in the quadrate, that looke how much the numerator 40, doth want of 2107, ſo much the cubicke giuen doth differ from the next cubick aboue it. And therefore if you ſubtract 40 the numerator, from 2107 the denominator, and ſhall adde the remaine 2067, vnto 17,616, the cubicke giuen, the whole ſhall be 19,686, the cubicke of the ſide 27.

The ſecond way is by parts of ſome great denomination, ſo as it be vnderſtood that they be cubicall parts, that their ſide may be certainly knowne before. As for example, the ſame number 17,616, reduced vnto one hundred cubicall parts, that is, vnto 1,000,000, do make 17,616,000,000, for the numerator. The parts then are thus; $\frac{17,616,000,000}{1,000,000}$. Now the ſide of the numerator is 2,601, for the numerator of the one hundred parts giuen: (For the former denominator being made by the multiplication of 100, by it ſelfe cubically; the roote or ſide of it muſt needs be 100, for the denominator of the parts ſought.) Therefore the parts deſired are $\frac{2,601}{100}$, that is, by reduction $26\frac{1}{100}$, and beſides that, there do remaine 19,712,199, which cannot adde ſo much as $\frac{1}{100}$ part vnto the ſide foūd; becauſe that the difference of this cubicke from the next greater aboue it, is greater then this remaine: And therefore that remaine is neglected, as not of any moment. Thus farre of the extraction or finding out of a cubicke ſide or roote.

10 *The two complements are the two meanes continually proportionall betweene the two cubes: that is, as the greater cube is vnto the greater complemenent; ſo is the greater complement vnto the leſſer: and ſo is the leſſer complement vnto the leſſer cube.*

As for example, Let the whole cubicke be 1728. Here the two ſeuerall cubes, let them be 1000, and 8. The greater complement let it be 200, and the leſſer 40, as we haue ſhewed at the *6*.p. Here I ſay 200 and 40 are the meane proportionals betweene the two cubes 1000 and 8: That is, as 1000 are to 200, ſo are 200 to 40; and ſo are 40 to 8. The cauſe of this is manifeſt by the 5 e xxij R, which teacheth, that, Solids that are alike, haue a trebled reaſon of their correſpondent ſides; And alſo they haue two meane proportionals comprehended of the croſſe multiplication of the baſe and height of the extremes: *19* p viij E. But the two complements here are contained of the baſe and height of the extremes croſſely multiplied. Therfore the two complements are the meane proportionals betweene the two cubes, that is, the two extremes.

11 *If foure numbers be continually proportionall, the products of the extremes by the each other ſquares, ſhall be the cubes of the middle numbers: to wit, the greater of the greater; and the leſſer of the leſſer. Iordanus* 57 p 6, *of his Arithmeticke.*

Let *16*, 24, 36, and 54, be foure numbers continually proportionall: And let the quadrate of *16*, be 256; of 54, be 29*16*. Againe, let the product of *16*, the leſſer extreme, by 29*16*, the ſquare of 54, the greater number, be 4*6*,*656*. Item, let the product of 54 the greater extreme, by 25*6*, the ſquare of *16* the leſſer extreame, be 13,824. Here I ſay, the ſide of 4*6*,*656*, the greater cubicke, ſhal be 3*6*: and the ſide of 13,824, the leſſer, ſhall be 24, as may be proued for experience, by the former. The example is thus:

The foure numbers giuen, *16* 24 3*6* 54
The ſquares of the extremes: *16* 54
16 54
25*6* 29*16*

The product of the greaters ſquare by the leſſer extreme, 2916
16
46656

The product of the leſſers ſquare by the greater extreame: 256
54
13824

Now the ſide of 46,656, the greater, is 36; of 13,824, the leſſer, it is 24. But 24, and 36, are the middle numbers of the foure number: Therefore, *If foure numbers be continually proportional, the products of the extremes by their each other ſquares, are the cubes of the meane.*

This rule then, as you ſee, is a kinde of *Meſolabium*, that is, a way to finde out the middle numbers, or meane proportionals: whereby I meane, the extremes being giuen, the meane proportionals are eaſily found.

Any two numbers are thus in the maner of ſolids alike betweene themſelues, conceiued, as oft as between them two meane continually proportional, are ſought. But many times theſe middle numbers, the cubicall ſides of the products of the extreames by the each other quadrates or ſquares, are ſurd numbers, as they call them, that is, ſuch as cannot be expreſſed by arithmeticall numbers, but by ſuch characters as are deuiſed by the Algebraiſts.

Many other rules might hither be added, but we eſpecially in this place regard ſuch as may any kinde of way help, either to the vnderſtanding of this preſent argument of extraction of the ſquare and cubicke roote; or may be of vſe for the making of our *Meſolabium architectonicum*, or Carpenters rular.

12. *The product made continuall of three numbers continually proportionall, ſhall be the cube of the meane or middlemoſt number.* 36 p 11 E.

Let 4, 6, 9, be three numbers continually proportionall: And let them be multiplied continually betweene themſelues: 216, the product ſo made, ſhall be the cube of 6, the middle number. This propoſition, ſaith Schoner, is a kinde of golden rule in ſolids, by which hauing the one of the extremes of the three proportionals giuen, with the ſolid made of them all multiplied betweene themſelues continually, the reſt are alſo giuen. As for example, Let 216 be the ſolid giuen, made after this order of three proportionall numbers, and let 4 be the one extreme giuen. Here I ſay, the middle number by this rule ſhall be 6. For the Cubicke roote of 216, is 6. Now the product of 4 by 6, is 24; and the quotient of 216, by 24, is 9, the third or greateſt extreame. Or, the product of 6 by it ſelfe, is 36. Now 36 diuided by 4, doth yeeld 9, for the other extreame. For, if three numbers be continually proportionall, the product of the middlemoſt by it ſelfe, ſhall be equall to the product of the two extreames by themſelues, by the rule of proportion. Therefore, if one of the extreames giuen ſhall diuide the product of the middlemoſt by it ſelfe, the quotient ſhall be the other extreame that was deſired.

13 *Hauing found the one of the meane proportionals, the other alſo ſhall be found, if the meane proportionall giuen be multiplied by the extreame that is fartheſt from it, the roote of the product ſhall be the meane proportionall deſired.*

As for example, Let 32, 16, 8, 4, be foure proportionals giuen; and ſuppoſe that the third bound were not knowne. Here therefore I multiply 16 by 4, the extreame that is fartheſt from it, and I make 64: Now the ſquare ſide of 64, by the 5 e j, is 8, the third proportionall ſought. Item, ſuppoſe that 16, the firſt of the two meane proportionals were vnknowne; Here 8 ſhall multiply 32, the firſt bound; not 4, the laſt; becauſe that is fartheſt off from it: The roote of 256, the product, by the e j, is 16. This roote is the firſt meane proportionall of the two, betweene 32 and 4, the extreames giuen.

CHAP. V.

Of the meaſuring of Timber by the Foote.

BY the Foote we do alſo meaſure Timber: but Timber being a ſolid body of three dimenſions, to wit, length, thickneſſe and breadth; by a foote of timber we vnderſtand here a cube of 12 inches ſquare, as they call it: (For here they abuſe the word, as alſo ſome euen of the learned haue done, as before we haue ſhewed:) That is, a foote of timber doth containe 1728 ſquare inches. Here therefore commonly two dimenſions are giuen, to wit, breadth and thickneſſe: the length is ſought. If then the ſquare timber to be meaſured be 12 inches thicke, and 12 inches broade; there is no queſtion of the length: For euery 12 inches of that peece ſhall make a foote of timber. But if the breadth and thickneſſe do varie neuer ſo little from theſe two caſes nominated by the ſtatute, although they varie no whit one from another, that is, although the breadth be equall to the thickneſſe, here preſently riſeth a queſtion, what the length ſhould be (according to that breadth and thickneſſe) that muſt make a foote ſquare, or that muſt be equall to that peece which is 12 in length, 12 in breadth, and 12 in thickneſſe.

Yet this euery one can doe, that knoweth ought in this buſineſſe: For the Carpenters haue vpon their Rulars, or vpon ſome peece of paper or parchment, all the ſquare meaſures ſet downe, from 1 inch ſquare, vnto 24, 36, or 40 inches ſquare. But if the breadth and thickneſſe be different one from another, whether little or much, this is not onely troubleſome and vnreadie, as the former, but very falſe and erronious, as ſome of their owne companie haue truly noted. For that of the ſquares, was made, as ſeemeth, by ſome skilfull in the Mathematicks; but their practiſe in the latter caſe, is deceitfull, falſe, and wholy againſt the rules of Geometrie, and can no way be iuſtified Now becauſe that all men that haue occaſion to vſe this skill of meaſuring, do not vnderſtand how theſe tables or rules are made: as alſo for that the ſame being copied out oft times by vnskilfull men; I thinke it not amiſſe here to ſet downe by the former grounds, as we haue done before for boord-meaſure and others, the maner of calculating and making of the ſame: and that eſpecially becauſe that theſe tables tend directly to the making of our Rular, which is the chiefe cauſe that firſt moued me to vndertake this labour, or to write of this argument. The Rule therefore whereby this is performed, is thus: *If by the product of the breadth and thickneſſe giuen, thou ſhalt diuide the cube of 12, (that is, 1728) the quotient ſhall ſhew the length required to make a foote of timber.* The formes of timber which commonly are to be meaſured of Carpenters, are long ſquares (ſuch as the Geometers call *Parallellepipida oblonga*) or Rounds, otherwayes called by them Cylinders, which the vulgar ſort of carpenters do alſo account, as we haue ſhewed, amongſt the number of ſquares, whoſe breadth and thickneſſe are the ſame. A foot ſquare (a cubicall foote I meane) ſaith our ſtatute, as we haue before taught, muſt containe 1728 ſquare inches. If therefore the breadth and thickneſſe be greater or leſſe then 12, the length ſhall be found by the preſcript of the former rule. Suppoſe, for example, that a peece of ſquared timber were to be meaſured, whoſe breadth and thickneſſe are equall, yet greater then 12, as namely 16 inches: Here I multiply the breadth by the thickneſſe, that is, 16 by 16; and I make 256. By this product I diuide 1728, and I find the quotient 6¾: Therefore I ſay, that euery 6 inches, and three quarters of one inch, ſhall, of that breadth and thickneſſe, make a foote of timber, according to the intention of that Statute.

Item, ſuppoſe the peece to be meaſured, whoſe breadth and thickneſſe is equall, were 6 inches ſquare, and it were demanded, how much of that in length would make a foot of timber? R. The product of 6 by 6, is 36: Now the quotient of 1728 by 36, is 48. Therefore I ſay, that 48 inches of that ſticke are required to make a foote of timber.

Now ſuppoſe the breadth and thickneſſe were vnequall; as for example, firſt let the breadth be 18, and the thickneſſe 12: here the product of 18 by 12, is 216; and the quotient of 1728 by 216, is 8: Therefore I ſay, euery 8 inches of that peece of timber ſhall be equall to a foote of ſolid meaſure.

Admit a planke or table were to be meaſured after the maner of timber meaſure, whoſe breadth is 36 inches, and thickneſſe 4: how much in length ſhall make a foote of ſolid timber? R. The product of 36 by 4, is 144. And the quotient of 1728, by 144, is 12, the deſired length. Thus then a Table for ſolid or timber meaſure is to be made, like vnto thoſe which we haue before ſhewed for boord and land-meaſure. Onely this is to be obſerued, that this table is onely of ſuch peeces as are ſquare, that is, of ſuch whoſe breadth and thickneſſe are equal;although it may alſo be extended & made for all;if need ſo require.

If beginning at an vnitie, and ſo aſcending vpward vnto the greateſt, thou ſhalt deuide 1728 *by the ſquare of any number aſſigned, and ſo forth, the ſeuerall quotients placed againſt the number giuen, ſhall make the table of timber meaſure.* As for example, The ſquare of 1 is 1; and the quotient of 1728 by 1, is 1728. Item, the ſquare of 2 is 4;and the quotient of 1728 by 4, is 432. Laſtly, the ſquare of 12 is 144, and the quotient of 1728 by 144, is 12, for the length deſired. The Table then for ſquare meaſure, that is, for the meaſuring of ſuch timber where the breadth and thickneſſe are equall, is thus.

B	L	B	L	B	L	B	L
1	144^{f} 0^{i}	10	1^{f} $5\frac{7}{25}$	19	$4\frac{284}{361}$	28	$2\frac{10}{49}$
2	36 0	11	1 $2\frac{34}{121}$	20	$4\frac{8}{25}$	29	$2\frac{46}{841}$
3	16 0	12	1 0	21	$3\frac{45}{49}$	30	$1\frac{69}{75}$
4	9 0	13	0 $10\frac{38}{169}$	22	$3\frac{69}{121}$	31	$1\frac{767}{961}$
5	5 $9\frac{3}{25}$	14	0 $8\frac{5}{7}$	23	$3\frac{141}{529}$	32	$1\frac{91}{128}$
6	4 0	15	0 $7\frac{51}{75}$	24	3	33	$1\frac{71}{121}$
7	2 $11\frac{13}{49}$	16	0 $6\frac{3}{4}$	25	$2\frac{478}{625}$	34	$1\frac{143}{289}$
8	2 3	17	0 $5\frac{283}{289}$	26	$2\frac{376}{676}$	35	$1\frac{503}{1225}$
9	1 9	18	0 $5\frac{1}{3}$	27	$2\frac{270}{729}$	36	1 [illegible]

The vſe of this Table is eaſie out of the former: onely this difference is to be obſerued, that becauſe plaines haue but two dimenſions, therfore one dimenſion giuen there was ſufficient for the finding out of the other vnknowne. But here, for that ſolids haue three dimenſions, two (to wit, breadth and thickneſſe) are required for the finding out of the third deſired. Yet now againe, ſeeing that here the ſaid two dimenſions giuen, are equall one to another; if you ſhall with either of the dimenſions giuen, enter the firſt columne of the Table, the columne on the right-hand ſhall yeeld the length deſired. As for example, Admit a timber ſticke to be meaſured were 4 inches ſquare, that is, were 4 inches thicke, and 4 inches broade; I demand, how much of that ſticke in length, ſhall be required to make a foote of ſolid meaſure. R. Here becauſe 4 is equall vnto 4, that is, for that the breadth and thickneſſe are equall, I enter the columne on the left-hand with 4, and I finde anſwering to it, in the columne on the right-hand, 9^{f}: Therefore I ſay, euery *9* foote in length, of that breadth and thickneſſe, ſhall make a foote of ſolid meaſure; The product, I ſay, made continually of 4, 4, and 108, ſhall be equall to the product continually made of 12, 12, and 12; that is, the product continually made of 4, 4, and 108, ſhall be 1728, thus:

$$\begin{array}{r} 12 \\ 12 \\ \hline 144 \\ 12 \\ \hline 1728 \end{array} \qquad\qquad \begin{array}{r} 4 \\ 4 \\ \hline 16 \\ 108 \\ \hline 1728 \end{array}$$

Item, ſuppoſe a timber ſticke to be meaſured were *16* inches ſquare. The columne on the right-hand anſwering to *16*, giueth $6\frac{3}{4}^{i}$: Therefore I ſay, euery *6* inches, and 3 quarters of an inch in length, of that breadth and thickneſſe, ſhall make a foote of ſolid meaſure.

Againe, admit it were 48 inches ſquare: what length ſhall be required to make a foot of ſolid meaſure? R. Here 48 is greater then any number contained in the columne of breadth, on the left-hand: Therefore I take 24, the halfe of 48, and with that entring the Table, I finde 3^{i} to anſwer for the length deſired. But here obſerue, 24 as it is but the halfe of 48^{i}, the breadth: ſo is it alſo but the halfe of the thicknes, which is ſuppoſed likewiſe to be 48^{i}. Therefore 24 is but the one quarter of both the aſſigned: Wherfore I ſay that 3^{i} in length, of that breadth and thicknes, ſhall containe 4 foote of ſolid meaſure.

Laſtly, admit a pillar were but halfe an inch ſquare: Here I finde no number amongſt the breadths of the left-hand ſo litle as $\frac{1}{2}$: Therefore I enter the Table with 1^{i}, the double of $\frac{1}{2}$, & I find for the length deſired 144^{f}. But here becauſe 1^{i}, the breadth, is twiſe ſo much as $\frac{1}{2}^{i}$, the breadth giuen; and is alſo twiſe ſo great as the thickneſſe, which is ſuppoſed to be equall to the breadth. Therefore 144^{f}, or 1728^{i} in length, ſhall containe but $\frac{1}{4}$ of a foot of ſolid meaſure: or euery 576^{f} in length, of that breadth and thicknes, ſhall be equall to the cube of 12^{i}.

Hauing finiſhed the former Table for timber-meaſure, & knowing it to be ſeruiceable in thoſe caſes onely where the two dimenſions giuen are equall one to another, I bethought me of a Table more generall, for the meaſuring of all ſorts of ſquared timber whatſoeuer. That this might be done, I doubted not; but that it might be done in few words, or within ſo little a compaſſe as here thou ſeeſt, I knew not vntill I had made triall: And hauing contriued it, in the maner of Pythagoras tables, into a triangular forme, like vnto that muſicall inſtrument which Swidas, as I remember, calleth *Trigonum muſicum*, I thought good to name it of the forme and vſe, *Trigonum architectonicum*, The Carpenters ſquire. Now that by this Table all thoſe foule and groſſe errors are auoided, which in meaſuring of timber are by the cōmon ſort of workmen committed, that is, that by this our Table the Carpenter may not onely meaſure all ſorts of ſquared timber moſt exactly and truly, but alſo more eaſily and ſpeedily then by any way commonly practiſed or publiſhed; beſides that, it is demonſtrable out of the grounds of theſe arts, euery man of meane vnderſtanding, that ſhall pleaſe to compare them, ſhall be able to teſtifie with me.

The making of this Table is in a maner the ſame with that of the former: the difference onely is this, that there 1728, the cube of 12, was diuided onely by the ſquare of euery ſeuerall number giuen: Here the ſame cube is to be diuided, not onely by the ſquare, that is, by the product of euery ſuch number by it ſelfe, but alſo by the products of euery ſuch ſeuerall number by any other aſſigned whatſoeuer. Now it is manifeſt, that the quotient thus found (which anſwereth vnto the third dimenſion ſought) is to be placed in the common angle or meeting of the columnes of the two dimenſions giuen, that is, of thoſe two numbers, by whoſe product this quotient was found.

Firſt therefore hauing made a right-angled triangle, I place the numbers giuen vpon the ſides without the ſame: Theſe two ranks of numbers anſwering (as we haue ſaid) vnto the two dimenſions giuen, the one of them at the top beginneth with the greateſt of the numbers giuen, and endeth at the right-corner with an vnitie: The other running along vnderneath the triangle, beginneth at the ſame right corner with the greateſt, and endeth with the leaſt. Theſe numbers alſo, for the better helpe and guide vnto the eye, may alſo be placed vpon the *hypotenuſa*, or ſlanting ſide of the triangle which is oppoſite vnto the ſaid right-corner. This being done, I begin either at the greateſt or leaſt numbers, it skilleth not whether, and I finde the numbers for the third dimenſion, as before is taught, which I place within the triangle in their ſeuerall columnes, as the factors ſhall appoint. As for example, beginning at the greateſt, I multiply 24 by 24; and with 576 the product, I diuide 1728, and I finde 3, which I place with in the triangle in the firſt columne againſt 24, &c. Againe, I multiply the ſame 24 by 23, and with the product 552, I diuide 1728, and the quotient $3\frac{72}{552}$ (or $3\frac{3}{23}$, being reduced vnto parts of the leaſt denomination) I ſet in the ſame firſt columne againſt 23. In like maner I multiply 23 by 23; and by the product 529, I deuide 1728, and I finde $3\frac{141}{529}$, which I place in the ſecond columne from the right-hand, againſt the ſame 23 on the outſide. Item, thus I multiply 22, firſt by 24; then by 23; and laſtly by 22; by the products ſtill diuiding the ſame 1728, I ſet the quotients in their ſeuerall places. Thus dealing with all the numbers on the right-hand of the Triangle, vntill thou comeſt vnto 1, the loweſt on that ſide, the Table ſhall be made.

The vſe of the Table in Timber-meaſure.

This Table, as the Title ſheweth, ſerueth generally for the meaſuring both of Plaines and Solids by the foote. Now Solids, ſuch as Timber and Stone are, haue three dimenſions, to wit, breadth, thickneſſe and length. Of theſe, two, breadth and thickneſſe, are giuen: The third, I meane length, is ſought. Here therefore, *Seeke the one of the dimenſions giuen amongſt thoſe numbers on the outſide of the Triangle, which at the top begin at 24, and ſo deſcend to 1: The other ſeek amongſt thoſe at the baſe vnderneath the ſame: The ſpace within the Triangle, to which theſe two do point, ſhall ſhew the length deſired.* As for example, Suppoſe a peece of ſquared timber were 12 inches broade, and 12 inches thicke: In the common angle, or meeting of theſe two numbers, next to the beuelling or ſlanting line of the Triangle, I finde 12 for the length deſired. Againe, ſuppoſe the breadth of ſome ſtone to be meaſured were 18 inches, and the thickneſſe 6. Theſe numbers do ſhew the length deſired to be 16 inches: Not 12, as the vulgar rules do teach.

In Boord-meaſure.

In boord-meaſure, onely breadth and length are conſidered: Therefore here, *The breadth given, is ſought ſometimes amongſt thoſe numbers on the outſide upon the right-hand: Sometimes amongſt thoſe underneath the ſame, as occaſion ſhall ſerve: For it muſt ſo be taken, that it may meet continually with* 12. An example or two ſhall make all plaine: Suppoſe a boord were *9* inches broade; here I ſeeke *9* amongſt thoſe numbers on the right hand: The common angle or meeting of this with 12, of thoſe numbers at the bottome, doth ſhew that 16 inches is the length deſired. Againe, ſuppoſe it were 18 inches broade; here no number on the right hand, greater then 12, can meete with 12, in the vnder-ranke: Therefore I ſeeke 18 amongſt thoſe of the lower ranke; And I finde the common angle of 18 of this rank, and of 12, of the numbers amongſt thoſe on the right hand, to giue 18 inches for the length deſired.

If any man ſhall deſire that the fractions ſhould be made more ſeruiceable for the vſe of practitioners, it may eaſily be done if any man will take the paines.

FINIS.

TRIGON

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A Table ſeru

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ONVM ARCHITECTONICVM:

OR,

E CARPENTERS SQVIRE:

that is,

ſeruing for the meaſuring of Boord, Glaſſe, Stone, Timber, and ſuch like Plaines and Solids *by the Foote.*

													24	
												23	3	24
											22	$3\frac{141}{529}$	$3\frac{3}{23}$	23
										21	$3\frac{69}{121}$	$3\frac{105}{253}$	$3\frac{3}{11}$	22
									20	$3\frac{45}{49}$	$3\frac{57}{77}$	$3\frac{93}{161}$	$3\frac{3}{7}$	21
								19	$4\frac{8}{25}$	$4\frac{4}{35}$	$3\frac{51}{55}$	$3\frac{87}{115}$	$3\frac{3}{5}$	20
							18	$4\frac{284}{361}$	$4\frac{57}{95}$	$4\frac{44}{133}$	$4\frac{28}{209}$	$3\frac{417}{437}$	$3\frac{15}{19}$	19
						17	$5\frac{1}{3}$	$5\frac{1}{19}$	$4\frac{4}{5}$	$4\frac{4}{7}$	$4\frac{4}{11}$	$4\frac{4}{23}$	4	18
					16	$5\frac{283}{289}$	$5\frac{11}{17}$	$5\frac{113}{323}$	$5\frac{7}{85}$	$4\frac{100}{119}$	$4\frac{116}{187}$	$4\frac{164}{391}$	$4\frac{4}{17}$	17
				15	$6\frac{3}{4}$	$6\frac{6}{17}$	6	$5\frac{14}{19}$	$5\frac{2}{5}$	$5\frac{1}{7}$	$4\frac{10}{11}$	$4\frac{16}{23}$	$4\frac{1}{2}$	16
			14	$7\frac{51}{75}$	$7\frac{1}{5}$	$6\frac{66}{85}$	$6\frac{6}{15}$	$6\frac{6}{95}$	$5\frac{76}{100}$	$5\frac{17}{35}$	$5\frac{13}{55}$	$5\frac{1}{115}$	$4\frac{4}{5}$	15
		13	$8\frac{40}{49}$	$8\frac{8}{35}$	$7\frac{5}{7}$	$7\frac{31}{119}$	$6\frac{3}{4}$	$6\frac{66}{133}$	$6\frac{6}{35}$	$5\frac{129}{147}$	$5\frac{45}{77}$	$5\frac{59}{161}$	$5\frac{1}{7}$	14
	12	$10\frac{38}{169}$	$9\frac{45}{91}$	$8\frac{68}{195}$	$8\frac{4}{13}$	$7\frac{181}{221}$	$7\frac{5}{13}$	$6\frac{246}{247}$	$6\frac{42}{65}$	$6\frac{30}{91}$	$6\frac{6}{143}$	$5\frac{233}{299}$	$5\frac{7}{13}$	13
11	12	$11\frac{1}{13}$	$10\frac{2}{7}$	$9\frac{3}{5}$	9	$8\frac{8}{17}$	8	$7\frac{11}{19}$	$7\frac{1}{5}$	$6\frac{6}{7}$	$6\frac{6}{11}$	$6\frac{6}{23}$	6	12

													13
12													10
11												12	11
10											$14\frac{34}{121}$	$13\frac{1}{11}$	12
9										$17\frac{7}{25}$	$15\frac{39}{55}$	$14\frac{2}{5}$	13
8									$21\frac{1}{3}$	$19\frac{1}{5}$	$17\frac{5}{11}$	16	14
7								27	24	$21\frac{3}{5}$	$19\frac{7}{11}$	18	16
6							$35\frac{13}{49}$	$30\frac{6}{7}$	$27\frac{3}{7}$	$24\frac{24}{35}$	$22\frac{34}{77}$	$20\frac{4}{7}$	18
5						48	$41\frac{1}{7}$	36	32	$28\frac{4}{5}$	$26\frac{2}{11}$	24	22
4					$69\frac{3}{25}$	$57\frac{3}{5}$	$49\frac{13}{35}$	$43\frac{1}{5}$	$38\frac{2}{5}$	$34\frac{14}{25}$	$31\frac{23}{55}$	$28\frac{4}{5}$	26
3				108	$86\frac{2}{5}$	72	$61\frac{5}{7}$	54	48	$43\frac{1}{5}$	$39\frac{3}{11}$	36	33
2			192	144	$115\frac{1}{5}$	96	$82\frac{2}{7}$	72	64	$57\frac{3}{5}$	$52\frac{4}{11}$	48	44
1		432	288	216	$172\frac{4}{5}$	144	$123\frac{3}{7}$	108	96	$86\frac{2}{5}$	$78\frac{6}{11}$	72	66
	1728	864	576	432	$345\frac{3}{5}$	288	$246\frac{6}{7}$	216	192	$172\frac{4}{5}$	$157\frac{1}{11}$	144	132
	1	2	3	4	5	6	7	8	9	10	11	12	13

13	8 40/49	8 8/35	7 5/7	7 31/119	6 3/4	6 66/133	6 6/35	5 129/147	5 45/77	5 59/161	5 1/7	14
10 18/169	9 45/91	8 68/195	8 4/13	7 181/221	7 5/13	6 246/247	6 42/65	6 30/91	6 56/143	5 233/299	5 7/13	13
11 1/13	10 2/7	9 3/5	9	8 15/34	8	7 33/57	7 1/5	6 6/7	6 6/11	6 6/23	6	12
12 52/143	11 17/77	10 26/55	9 52/13	9 45/187	8 8/11	8 56/209	7 47/55	7 111/231	7 17/121	6 210/253	6 6/11	11
13 19/65	12 12/35	11 13/25	10 4/5	10 14/85	9 3/5	9 9/95	8 16/25	8 8/35	7 47/55	7 59/115	7 1/5	10
14 10/13	13 5/7	12 4/5	12	11 5/17	10 2/3	10 2/19	9 3/5	9 1/7	8 8/11	8 8/23	8	9
16 8/13	15 3/14	14 2/5	13 1/2	12 12/17	12	11 7/19	10 4/5	10 2/7	9 9/11	9 9/23	9	8
18 89/91	17 31/49	16 16/35	15 3/7	14 62/119	13 5/7	12 132/133	12 1/5	11 111/147	11 17/77	10 118/161	10 2/7	7
22 2/13	20 4/7	19 1/5	18	16 16/17	16	15 9/57	14 2/5	13 5/7	13 1/11	12 12/23	12	6
26 38/65	24 24/35	23 1/25	21 3/5	20 28/85	19 1/5	18 18/95	17 7/25	16 16/35	15 39/55	15 3/115	14 2/5	5
33 3/13	30 6/7	28 4/5	27	25 7/17	24	22 14/19	21 3/5	20 4/7	19 7/11	18 18/23	18	4
44 4/13	41 1/7	38 2/5	36	33 15/17	32	30 6/19	28 4/5	27 3/7	26 2/11	25 1/23	24	3
66 6/13	61 5/7	57 3/5	54	50 14/17	48	45 9/19	43 1/5	41 1/7	39 1/11	37 13/23	36	2
132 12/13	123 3/7	115 1/7	108	101 11/17	96	90 18/19	86 2/5	82 2/7	78 6/11	75 3/23	72	1
13	14	15	16	17	18	19	20	21	22	23	24	

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